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WHAT ARE THE BEST HIERARCHICAL ORGANIZATIONS FOR THE SUCCESS OF A COMMON ENDEAVOUR?

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Abstract

All the human organizations are not governed by competition, power relationships and individual interest. Sometimes well-meaning people willingly decide to join their forces for a common project. The success of each participant, musicians of an orchestra, members of a eight rowing boat team, or of a danse troupe, is then integrally determine by the joint success. Besides the individual qualities of each member, the common success critically depends on the ability of the organizational structure to facilitate the flow of informations and orders between participants, i.e. to synchronize their individual actions.

Here, in the framework of an oversimplified mathematical modelization of the individual behavior, we investigate the synchronization properties of some typical hierarchical organizations and perform the comparison with existing ones. We show that the democratic network is the most stable one. One of our most surprising results concerns the existence of evolutionary culs-de-sacs, i.e. hierarchical structures that, although not optimal from the point of view synchronization, are not able to improve themselves under the effect of small perturbations.

1 Introduction

Hierarchy formation as a self-organization phenomenon

Hierarchical organizations are ubiquitous in animal and human societies [1, 2, 3, 4] although their origin is still an open issue [5]. Thanks to the pioneering works of Bonabeau [6] and the numerous studies that have followed in their wake [7, 8, 9, 10, 11], the nature and dynamics of the hierarchical pattern formation is well understood. Also it is now well established that individual differences between agents (like weight, size, age, talent or charisma) can not be responsible alone for the hierarchies observed in animal and human societies [12, 13] and that hierarchy formation is mainly a self-organization phenomenon due to social dynamics [14, 15, 16]. However understanding how hierarchical organizations spontaneously occur does not really help to understand why.

Selfish and altruist organizations

Among the recent studies investigating the mechanisms at the origin of hierarchical pattern formation, two diametrically opposed approaches can be identified. One method, the microscopic one, is concerned with selfish organizations where each agent is motivated by his own interest. Starting with the description of each agent's behavior, one investigates the consequences of the individual strategy onto the global organization of the connection network [17, 18]. On the contrary the other approach assumes that all agents are full of goodwill and that they ignore their individual satisfaction in favor of the success of a joint project [19]. The question is then how to organize the agents' relationships at a macroscopic level in order to maximize their collective action. Of course, such pure philanthropic organizations are clearly utopian and even volunteers in charitable associations, musicians in a philharmonic orchestra or members of a eight rowing boat team, may experience personal ambitions. However between these two extremes, selfish and altruist, lies the whole diversity of human organizations and the more recents publications take into account both aspects [20].

Modelization of an altruist organization

We will focus here onto the study of an idealized altruist organizations composed of almost identical agents. By almost identical agents, we intuitively want to exclude organizations like surgical intervention team where all is made to assist and facilitate a single agent's work, the one of the surgeon, and also to exclude situations where the final hierarchical position of an agent depends on its dynamics at $t = 0$ (i.e. no inheritance). Mathematically it means we restrict ourselves to situations where the master or slave character of any pair of agents, is solely determined by their coupling strengths but neither by their intrinsic difference nor by their initial dynamics.

As each agent does its best, our main simplifying hypothesis consists in assuming that the unique obstacle to the perfect functioning of the organization is a disagreement, misunderstanding or desynchronisation between the agents. Then a possible modeling (supplementary material A) consists in considering each agent as an oscillator standing on a network node, and to mimic the interactions between two collaborators as a coupling link between two neighboring nodes [19]. The coupling strength is then exactly the weight of the directed link, and can be positive or negative depending on whether the slave is forced to be locked in phase or in anti-phase with its master. Because of the recurrent nature of the dynamics of an oscillator, initial conditions play no role in the long time regime. Then the globally synchronized solution, where all the agents are locked in phase, is expected to correspond to the perfect functioning of the society. After a disturbance, the ability of the organization to return as soon as possible to the synchronized state is measured by the largest real part of the eigenvalues of the linear stability analysis. The more negative the largest eigenvalue real part, the smaller the resilient time (supplementary material B). Finally, the comparison between the various hierarchical organizations will not be based onto their ability to produce something or to perform a given work, but to their ability to resist to a perturbation (supplementary material C).

Note that, if at first sight this deliberate choice of an over simplified modelization may seem surprising, especially in the field of human sciences, it is certainly not new: in case of selfish human society, the paradigmatic prisoner's dilemma, simplistic to the point of being almost caricatural, has nevertheless been proved to be very useful and to lead to predictions remarkably realistic [17, 18]. Here we just apply this over simplification approach to the case of altruist human society.

Many diverse and varied oscillators have been considered in the literature, even chaotic ones. Because we are interesting in exact analytic results, we will restrict ourselves to the Kuramoto's ones [21], either because they are just the simplest linear oscillator one can imagine or because they are considered, since a very long time, as a simple paradigm for synchronization phenomena [19]. They are described as

$$\partial_t \theta_i = \omega_i + \sum_j A_{ji} sin(\theta_j - \theta_i) \tag{1}$$

where θ_i is the phase of oscillator i, ω_i its natural frequency and A_{ji} stands for amplitude of the force exerted by j on i. As we are interesting in maximizing the stability through an optimization of the configuration network and not through an increase of the average coupling strength, we will require the total mass of the links to be constant, and without loss of generality, to be equal to unity

$$mass(A) = \sum_{j,i} A_{j,i}^2 = 1 \tag{2}$$

The existence of such a physical upper limit for the coupling strength is not surprising. It is both a mathematical necessity to avoid divergences in the optimization process, but also a physiological reality for the creation and maintenance of a connection costs time and energy. Now other definitions of $mass(A)$ could have been possible, but the one we selected will turn out to strongly simplify our computations [22].

Two agents

The illustrative case of 2 agents is now studying in order to clarify the notations and the simplifying assumptions. The dynamic is expressed as

$$\partial_t \theta_1 = \omega_1 + A_{21} sin(\theta_2 - \theta_1) \qquad \partial_t \theta_2 = \omega_2 + A_{12} sin(\theta_1 - \theta_2) \tag{3}$$

with $A_{12}^2 + A_{21}^2 = 1$. Looking for the synchronized solution as

$$\theta_1(t) = \Omega t + \phi_1 \qquad \theta_2(t) = \Omega t + \phi_2 \tag{4}$$

we end up with

$$\Omega = \frac{A_{12}}{A_{12}+A_{21}}\,\omega_1 + \frac{A_{21}}{A_{12}+A_{21}}\,\omega_2 \text{ and } sin(\phi_2 - \phi_1) = \frac{\omega_2 - \omega_1}{A_{12}+A_{21}} \tag{5}$$

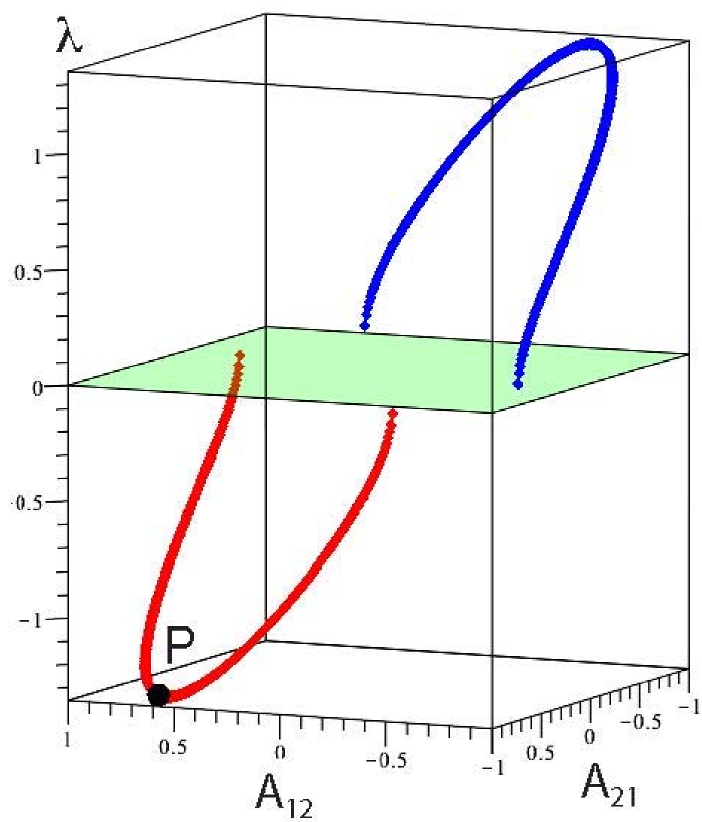

Figure 1: Plot of the linear stability eigenvalue λ_2 versus A_{12} and A_{21}. The blue (red) points correspond to an unstable (stable) synchronized solution. The discontinuities near the $\lambda = 0$ plane correspond to values of the coupling strengths (A_{12}, A_{21}) for which the synchronized solution does not exist. P stands for the most stable synchronized solution.

Several remarks are in order:

1. The limiting case $A_{12} = 0$, $A_{21} \neq 0$ and $\Omega = \omega_2$ (resp. $A_{21} = 0$, $A_{12} \neq 0$ and $\Omega = \omega_1$) corresponds to the situation where the dynamics of the second oscillator (Θ_2) can't be influenced by the first

one (Θ_1) since there is no flux of information from (Θ_2) toward (Θ_1). On the contrary, as A21 $\neq 0$, the dynamics (Θ_1) is sensitive to those of (Θ_2). In such a situation, (Θ_1) is a pure slave and (Θ_2) a pure master. Between these two extremes, there exist a continuous range of behaviors where the agents mutually influence each other.

2. The existence of a synchronized solution requires $|\omega_2 - \omega_1| < |A_{12} + A_{21}|$, i.e. a high enough coupling strength ($|A_{12} + A_{21}|$) to compensate the individual intrinsic differences ($|\omega_2 - \omega_1|$). As intended by our modelization, the sign of $\omega_2 - \omega_1$ plays no role which is the proof of the absence of an intrinsic predisposition to a slave or master behavior.

3. The existence of the synchronized solution does not necessary imply its stability with respect to small perturbations.

To investigate the linear stability of the synchronized solution, we slightly perturb it

$$\theta_1 = \Omega t + \phi_1 + \varepsilon\alpha_1(t) \qquad \theta_2 = \Omega t + \phi_2 + \varepsilon\alpha_2(t) \tag{6}$$

where $\varepsilon << 1$. At first order ε^1, we get

$$\partial_t \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = \underbrace{cos(\phi_2 - \phi_1)\begin{pmatrix} -A_{21} & A_{21} \\ A_{12} & -A_{12} \end{pmatrix}}_{M} \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} + O(\varepsilon^2) \tag{7}$$

The eigenvalues of M are

$$\lambda_1 = 0 \qquad \lambda_2 = -cos(\phi_2 - \phi_1)(A_{12} + A_{21}) \tag{8}$$

The vanishing eigenvalue (λ_1) is not relevant since it is associated with the continuous phase symmetry ($\phi_{1,2} \to \phi_{1,2} + \delta\phi$). The second eigenvalue λ_2 is real and is sign controls the stability of the synchronized solution (fig.1). Among the various possible coupling strengths (A_{12}, A_{21}) corresponding to a stable synchronized solution, there is one which is more resistant than the other. It correspond to $A_{12} = A_{12} = 1/\sqrt{2}$ (point P in fig.1). In this configuration, the two agents are not only perfectly synchronized, but also the time need to recover a synchronized state after a perturbation (the resilient time) is the smallest possible one.

Hierarchical organization as a cultural product

Knowledge of hierarchical organizations viable and proven belongs to our culture and as such, can be learned and transmitted. The most stable organizations, i.e. those which have proved more resistant to disturbance and survived the longest, will naturally be the most imitated by later generations. Also from one generation to another, small modifications in the hierarchical organizations that spontaneously occur, can be perpetuated when they lead to an obvious improvement of the group functioning. Hence natural selection is at work for hierarchical organizations and provides a slow variational dynamics where the fitness is the resilient time of the synchronized solution. For example, in the illustrative framework of the two agents case, fig.1 tell us that there are no topological obstacle for small perturbations of the coupling strengths to continuously drive the P' configuration up to the optimal P one.

Objectives

In the mathematical framework hence defined (i.e. non identical Kuramoto's oscillators, constant mass of the adjacency matrix and evolutionary dynamics), we are going to analytically prove the existence of cul-de-sacs where the natural selection is blocked. In these kind of configurations, any *generic* small perturbations of the connection network can be shown to drive the organization in a less stable regime. The possible small perturbations which may drive the organization toward a more stable state are so highly improbable that only strong perturbations (i.e a revolution) can make things evolve. Once the existence of cul-de-sacs proved, then one is naturally confronted with the question of the existence and nature of the optimal configuration, the one which would maximize the linear stability. In the case of identical Kuramoto's oscillator, we analytically show that the democratic configuration, where each agent is connected to all the others with the same positive strength is the most stable. When dealing with slightly non identical oscillators, a perturbative computation shows that the optimal configuration has almost the same linear stability as the homogeneous case, but generically displayed negative interactions [23].

2 Evolutionary culs-de-sacs

Sensitive and nonsensitive networks

Recently it has been reported that networks with the same number of nodes, same number of links (which implies same mass), and identical eigenvalues of the coupling matrix can exhibit fundamentally different approaches to synchronization depending on the degeneracy of associated eigenvectors [24, 25]. In contrast to sensitive networks (degenerated), nonsensitive networks (not degenerated) are predicted and experimentally observed to be more robust against parameter perturbations, what is called structural stability in dynamical system theory. We now re-investigate this aspect in the framework of Kuramoto's oscillators, but with an evolutionary point of view.

The synchronized solution of (1) is expressed as

$$\theta_i(t) = \Omega t + \phi_i \qquad \Omega - \omega_i = \sum_j A_{ji} sin(\phi_j - \phi_i) \tag{9}$$

Performing a linear stability analysis $\theta_i = \Omega t + \phi_i + \varepsilon \psi_i$ where $\varepsilon << 1$, we are left with

$$\partial_t \psi_i = \sum_j A_{ji} cos(\phi_j - \phi_i)(\psi_j - \psi_i) \tag{10}$$

Fig.2 deals with $N = 6$ identical Kuramoto's oscillators, with 5 links (for the sake of clarity, their mass has not been normalized and is then equal to 5). It displays 5 configurations with exactly the same spectrum, i.e they have the same set of linear eigenvalues. They are said to be iso-spectral. In itself, this observation is not exceptional since it has been shown long time ago by A. Schwenk that "Almost all trees are co-spectral" [26]. From the point of view of the dynamical stability alone, all these fives configurations converge exponentially toward the synchronized solution as e^{-t} $P_m(t)$ where m, the order of the polynomial P_m, is the degeneracy of the jordan block.

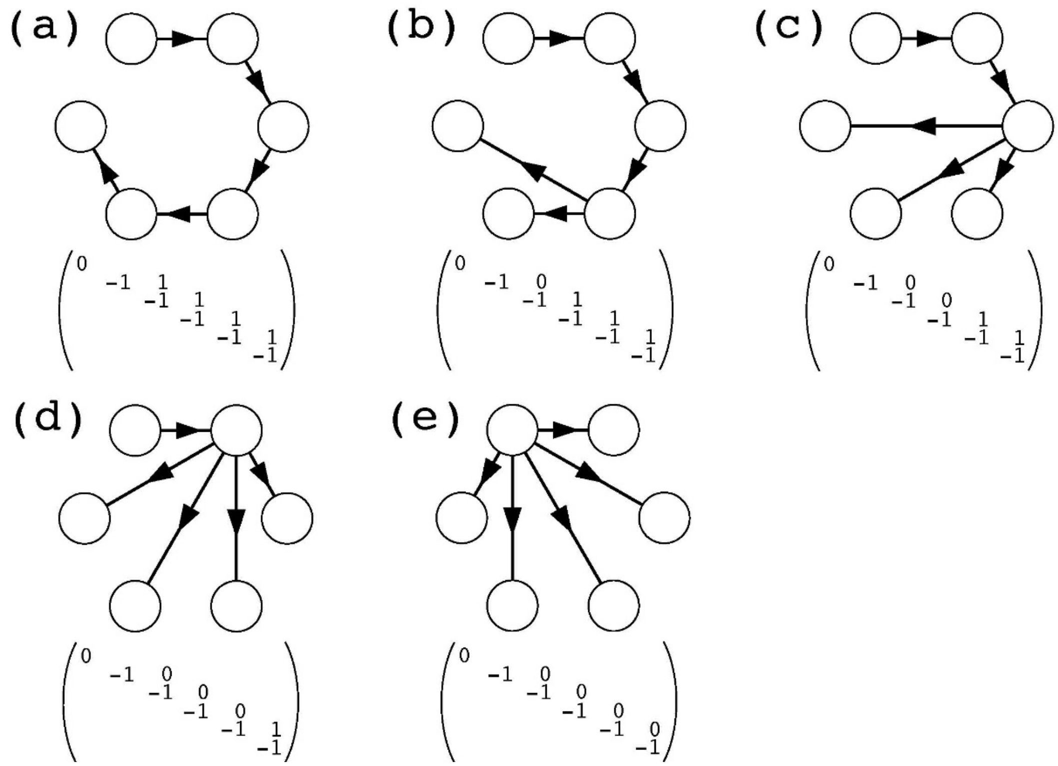

Figure 2: Five graph configurations with 6 nodes and 5 links are considered. For each graph, the top plot displays the topology of the link, while the bottom correspond to the jordan form of the Laplacian matrix. All the configurations are iso-spectral and only differ by the number of 1 in the first sub-diagonal of their jordan matrix.

In order to investigate the structural stability, the configuration networks of fig.(2) are now slightly perturbed

$$A_{j,i} \to A_{j,i} + \varepsilon p_{j,i} \qquad \sum_{j,i} A_{j,i}^2 = \sum_{j,i} \left(A_{j,i} + \varepsilon p_{j,i}\right)^2 \tag{11}$$

and $p_{i,i} = 0$. Simple analytical computations of the linear stability eigenvalues lead then to the following Taylor expansions, in which P_5 is a generic five degree polynomial in λ_1:

case(a)	$\lambda = -1 + \lambda_1 \varepsilon^{1/5} + \dots$	$\lambda_1^5 = Q_5$	$Q_5 = p_{6,1} - p_{6,2}$
case(b)	$\lambda = -1 + \lambda_1 \varepsilon^{1/4} + \dots$	$\lambda_1^4 = Q_4$	$Q_4 = Q_5 + p_{5,1} - p_{5,2}$
case(c)	$\lambda = -1 + \lambda_1 \varepsilon^{1/3} + \dots$	$\lambda_1^3 = Q_3$	$Q_3 = Q_4 + p_{4,1} - p_{4,2}$
case(d)	$\lambda = -1 + \lambda_1 \varepsilon^{1/2} + \dots$	$\lambda_1^2 = Q_2$	$Q_2 = Q_3 + p_{3,1} - p_{3,2}$
case(e)	$\lambda = -1 + \lambda_1 \varepsilon^{1/1} + \dots$	$P_5(\lambda_1) = 0$	

(12)

Highly non-monotonic, cusp-like dependence of the stability on the number of nodes and links of the network

Previous equations (12) have 2 very deep consequences, one has already been predicted [23, 24], the other is still unknown. First, the perturbed eigenvalues are continuous function of ε. Therefore in the regime of parameters where our perturbative analysis is valid ($\varepsilon << 1$), the real part of λ will never change its sign. However, although continuous, the derivative of the eigenvalues with respect to ε diverge to infinity when ε goes to zero. This means an infinite susceptibility! It is the reason why the configurations in fig.2 (arranged in order of decreasing susceptibility), although iso-spectral, are so different from the point of view of structural stability, and also the reason why, during an optimization process, one can observed highly nonmonotonic, cusplike dependence on the number of nodes and links of the network [23].

For the sake of clarity, we will not consider all the degenerated configurations of fig.2, but focus only on the case (a). However our reasoning, with only straightforward modifications, is easily generalized to the other cases. For the configuration (a) (that we will call from now "military"), the first correction satisfies $\lambda_1^5 = Q_5 = p_{6,1} - p_{6,2}$. For a generic perturbation Q_5 is not vanishing, the degeneracy is lifted and the unperturbed eigenvalue blows up into 5 pieces uniformly distributed around it. Hence, whatever the sign of Q_5, there are always at least one new eigenvalues with a real part larger than the unperturbed one. Therefore, we obtain that under the action of any small generic perturbation, the military hierarchy decreases its stability! This results have been clearly observed experimentally with a network of 4 optoelectronic feedback loops, and the critical role played by jordan bloc fully demonstrated [24]. Here the use of Kuramoto's oscillators instead of optoelectronics feedback loops, by simplifying the dynamics, allows exact analytical computations.

Consequences onto the evolutionary dynamics

The second deep consequence deals with evolution. Natural selection is the key mechanism by which biological traits become either more or less common in a population depending on their reproductive process. A random perturbation is "accepted" only if it leads to an increase of the fitness. In 2004, E.A. Variano, J.H. McCoy and H. Lipson [27] make use of genetic algorithm to generate a large number of highly stable networks, hence introducing the idea that networks could be submitted, as animal species, to natural selection. They based their fitness function on the number of eigenvalues with negative real part. Here we identify the highest eigenvalue real part as our fitness function, i.e. as the criteria that must be optimized during the evolution. We make this choice because we want our evolution dynamics to be able to distinguish between networks with the same number of negative eigenvalues but with different eigenvalues. Then, from the point of view of natural selection, configuration a) in fig.2 is an evolutionary cul-de-sac: not an ecological niche with a local maximum fitness, but a labyrinthine dead-end whose output, which does exist, is particularly difficult to reach. Indeed as the network possesses $N=6$ nodes, the perturbation $p_{i,j}$ lives in a $N(N-1)-1$ (=29) dimensional space. If we want this perturbation to be able to drive the network into a more stable configuration, then we must at least require that Q_5 is vanishing. The dimension of the available remaining perturbation space is still gigantic $N(N-1)-2$ (=28), but it is as difficult to find a 28 dimensional space inside a 29 dimensional one, than to find a point in a line, or a line in a plane.
However this is not the end of the story. Let us go on and restrict ourselves to perturbations such that $Q_5=0$. Then simple analytical manipulation leads to

$$Q_5 = 0 \Rightarrow \begin{cases} \lambda = -1 + 0\varepsilon^{1/5} + \lambda_2 \varepsilon^{1/4} + \ldots \\ with \begin{cases} \lambda_2 = 0 \\ or \\ \lambda_2^4 = \zeta_5 = p_{5,2} - p_{5,1} + p_{6,3} - p_{6,1} \end{cases} \end{cases} \tag{13}$$

and then the same reasoning as before holds: as soon as ζ_5 is non vanishing, the unperturbed eigenvalue blows up in 4 pieces uniformly distributed around it, such that the linear stability is thereby necessarily reduced. This stability decreasing occurs for any generic perturbation in the already reduced $Q_5 = 0$ perturbation space. At this stage, a suitable perturbation, i.e. that could be able to increase the linear stability, has to be chosen in 27 dimensional space with $Q_5 = 0$ and $\zeta_5 = 0$, and is therefore as probable as to find a point in a plane by chance. The same reasoning can be continued until all the degeneracy is lifted. We then obtain that the larger the degeneracy the deeper the cul-de-sac! Hence configurations in fig.2 are arranged in descending order for time needed for evolution to improve their stability. The fact that many human associations are organized according to the configurations (a,b,c or d) is usually explained as a consequence of the individual node differences. We just have proved that it could be also the case because of the cul-de-sac effect which blocks their evolution. Finally note that, among the configurations displayed in fig.2, only the "philharmonic orchestra" organization (e) is not a cul-de-sac. It is therefore the most adaptive, the one which is the most susceptible to improve itself with random modifications of the network topology.

3 Optimal hierarchical organization

Of course, the next question is: what are the global minima, the genuine fix evolutionary points? We first answer in the case of identical Kuramoto's oscillators proving that the democratic network where each node is connected to all other with the same weight, is a global optimum, and then investigate the case of slightly non identical ones.

Identical agents

The adjacency matrix of the democratic network is expressed as $A^d_{i,j} = \xi(1-\delta_{i,j})$ where ξ = $(N(N-1))^{-12}$ is used for mass normalization. Because the Kuramoto's oscillators are identical, the general linear stability analysis (10) can be rewritten as

$$\partial_t \psi_i = \sum_j A^d_{ji}(\psi_j - \psi_i) = \sum_j L^d_{ij}\psi_j \tag{14}$$

The eigenvalues of the laplacian matrix L^d are then computed through straight algebraic computations. Beside 0 which is always an eigenvalue because of the phase invariance symmetry, there are $(N-1)$ eigenvalues all non degenerate and equal to $-\xi N$, such that the trace of L^d is $-(N-1)\xi N$. Now let us consider a new network defined by its adjacency matrix $A^{'} = A^d + P$, where P be a square matrix (P is not necessary small!), with a vanishing diagonal. Because of the mass conservation $mass(A^{'}) = mass(A^d + P) = mass(A^d)$, we have

$$\sum_{j,i} P_{j,i} = -\frac{1}{2\xi}\sum_{j,i} P^2_{j,i} < 0 \tag{15}$$

Eq.(14) is now expressed as

$$\partial_t \psi_i = \sum_j (A^d_{ji} + P_{ji})(\psi_j - \psi_i) = \sum_j L^{'}_{ij}\psi_j \tag{16}$$

The trace of $L^{'}$ satisfies

$$Tr(L^{'}) = Tr(L^d) - \sum_{i,j} P_{i,j} > Tr(L^d) \tag{17}$$

which necessarily implies that at least one eigenvalue has a real part higher than $-N\xi$ and proves that the democratic configuration is a global optimum. Note that the constraint on the mass is a key point of the proof (15) and an a posteriori justification of our definition of the $mass(A)$.

Slightly non identical agents

Now we investigate the case of slightly non identical Kuramoto's oscillators which are described by Eq.(9) for the synchronized solution (ϕ_i) and by (10) for its linear stability (ψ_i). Rewriting this later as

$$\partial_t \psi_i = \sum_j A'_{ji}\left(\psi_j - \psi_i\right) \quad \text{with} \quad A'_{ji} = A_{ji} cos\left(\phi_j - \phi_i\right) \tag{18}$$

highlights the strong formally analogy with eq.(14). The only difference lies in the conservation of mass. Here it is $\sum_{j,i} A_{ji}^2 = 1$ which is conserved and not $\sum_{j,i} A_{ji}^{'2}$. However

$$\begin{aligned} \sum_{j,i} A_{ji}^{'2} &= \sum_{j,i} A_{ji}^2 \left(1 - sin\left(\phi_j - \phi_i\right)^2\right) \\ &\approx 1 + O(\delta\omega^2) \end{aligned} \tag{19}$$

where $\delta\omega$ stands for the standard deviation of the ω_i distribution. Therefore, in case of slightly non identical oscillators, the optimization process is expected to lead to almost identical A'_{ji} weights and to eigenvalues close to $-\xi N$. A side result is that the common pulsation ω of the optimal synchronized solution is very close to the ω_i average

$$\omega = \frac{1}{N}\sum_i \omega_i + \underbrace{\sum_{j,i} A'_{ji} tan\left(\phi_j - \phi_i\right)}_{\approx \xi \sum_{j,i} tan\left(\phi_j - \phi_i\right);0} \tag{20}$$

To confirm these predictions, numerical simulations have been performed. We proceeded in the following way. First, the pulsations ω_i were uniformly randomly chosen in the range $[1 - d\omega, 1 + d\omega]$, $d\omega << 1$. Then, the elements of the initial adjacency matrix were uniformly randomly chosen in the range $[-1, +1]$ and next multiplied by a normalization factor to set the mass to unity. Finally random perturbations of the network (that conserve the mass and with an amplitude that decreases with time) were repeatedly generated and the new configurations were retained only if the linear stability is increased. Along this Monte-Carlo dynamics, we recorded the real part of the less stable eigenvalue ($Sup(\Re e(\lambda))$) together with the variance of the A'_{ji} ($V =$

$< A_{ji}^{'2} > - < A_{ji}^{'} >^2$). In fig.3, the numerical trajectories in the $(Sup(\Re e(\lambda), V)$ plane, corresponding to several $d\omega$ values, are in very good agreement with our predictions: the variance decreases toward zero and the real part of the less stable eigenvalue does converge toward a value very close to those obtained in the case of identical oscillators (marked as λ_d in Fig.3). Also, beside the existence of negative optimal interactions [23], we also observe a strong tightening of the optimized final distribution of the A_{ji}^{opt} around two narrow peaks of opposite sign (Fig.4a). Separating the positive weights from the negative ones $A_{ji}^{opt} = A_{ji}^{+} + A_{ji}^{-}$ ($A_{ji}^{+} > 0$ and $A_{ji}^{-} < 0$) as in fig.4b and c, we obtain a splitting of the optimized configuration into two networks: the one with positive weights is made of two disconnected subnetworks, while the other, with negative weights is bipartite (the two parts corresponding to the two previous disconnected subnetworks). It means that the oscillators which belong to the left graph in fig.(4.b) have almost all the same phase $\approx \phi_L$, the oscillators of the right graph share the almost same phase $\approx \phi_R$, and that there is a phase opposition between the two disconnectd graphs, i.e. $\phi_R - \phi_L \approx \pi$.

The previous result display strong analogy with is called the hipster effect [28]. In Social science, economics and finance, large ensemble of interacting individuals taking their decisions either in accordance (mainstream) or against (hipsters) the majority, are ubiquitous. Yet, trying hard to be different often ends up in hipsters consistently taking the same decisions, in other words all looking alike.

4 Conclusion

In conclusion, by restricting ourselves to coupled oscillators with an extremely simple dynamic, we have obtain exact analytical results dealing with the optimization toward higher stability of a set of Kuramoto's oscillators with respect to the topology of the connection network. Global optimal configurations for identical and slightly non identical oscillator's have been identified as well as Cul-de-sac of the evolution. The latter are not local minima of the optimization dynamics and therefore not ecological niches, but amazing configurations where the natural selection mechanism is statistically blocked.

The analytical results are as rigorous as their interpretation in terms of human society is contestable, but very tempting. Even if Kuramoto's model is long time ago considered as a simple paradigm for synchronization phenomena, it must be recognize that it only provides an

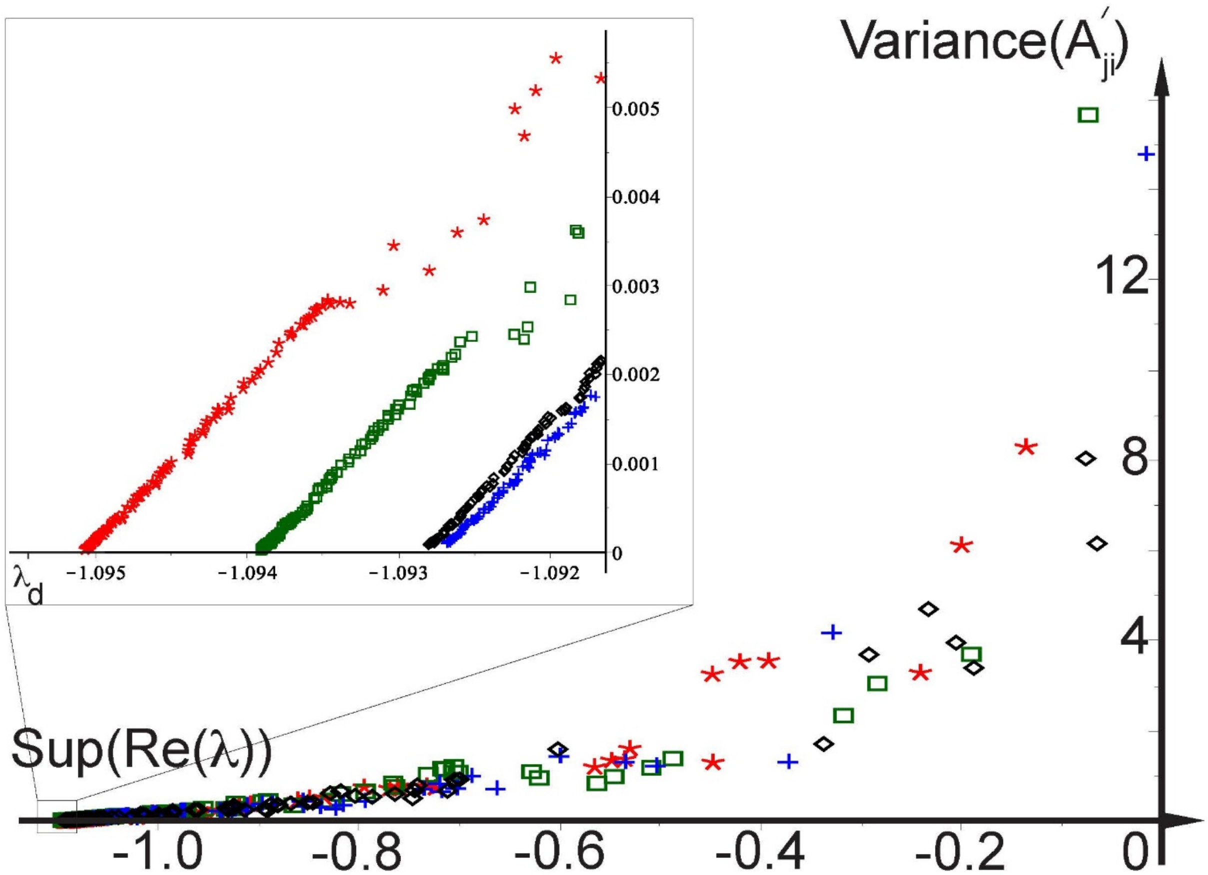

Figure 3: Monte-Carlo computations of the optimized network configuration in case of 6 nodes and adjacency matrix with unitary mass. The plot displays 4 trajectories in the (Sup$(\Re e(\lambda))$, Variance(A'_{ji})) plane for various ω_i distributions: asterics stand for ω_i $\in$ $[0.95,1.05]$, squares for $[0.93,1.07]$, diamonds for $[0.90,1.10]$ and crosses for $[0.87,1.13]$. The figure on the top left corner is a high magnification of the small one on the bottom left.

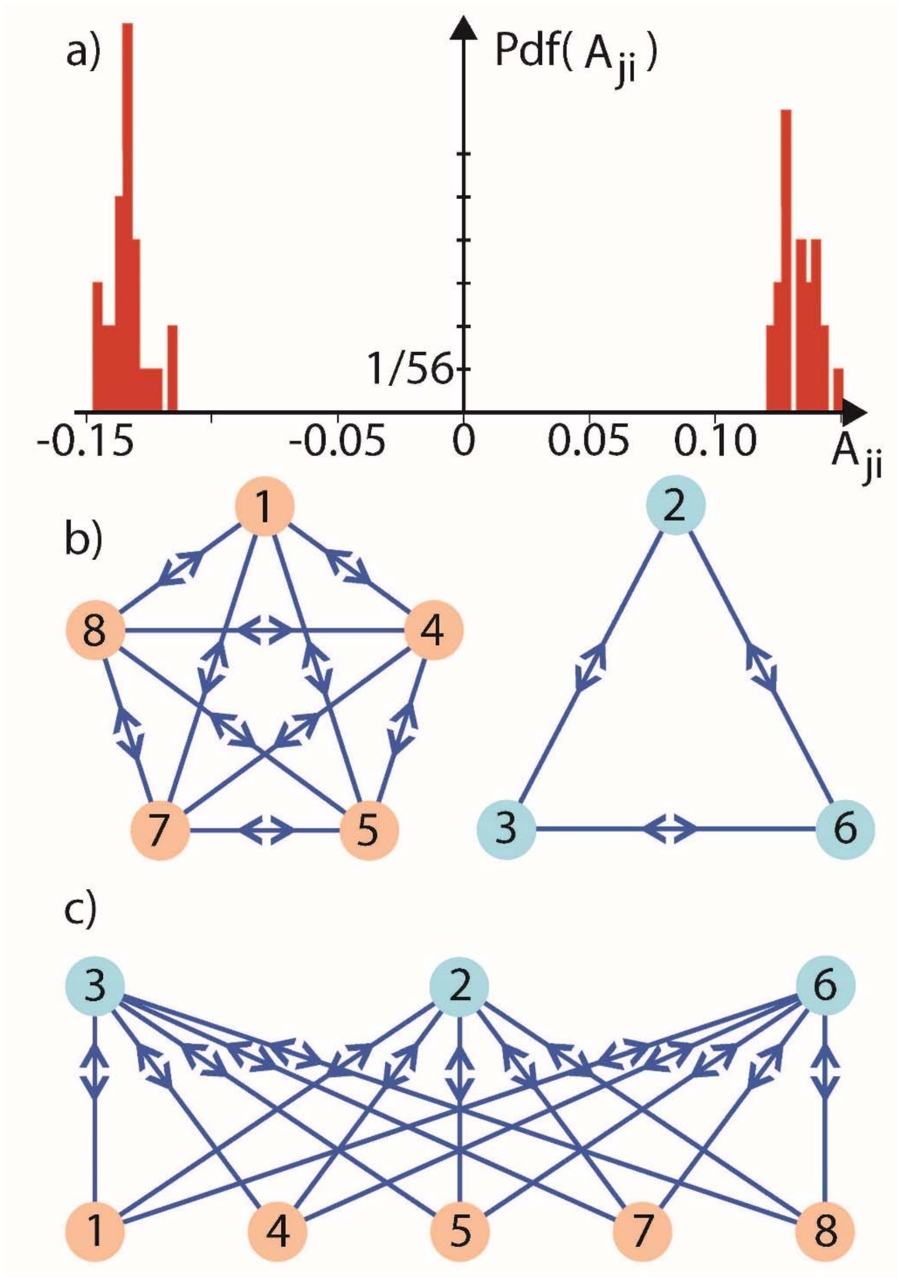

Figure 4: Optimized network of 8 non identical nodes and adjacency matrix with unitary mass obtained through a Monte-Carlo computation. The frequencies $\omega_{i\in[1,8]}$ are equal to $[0.9121, 0.9167, 0.9184, 0.9223, 1.0042, 1.0151, 1.0156, 1.0421]$ and the index i in ω_i corresponds to their position in the list. a) is the pdf of weights of the optimized adjacency matrix A_{ji}^{opt}, b) is the graph of the positive part of A_{ji}^{opt} (disconnected) while c) is associated with its negative part (bipartite).

oversimplified description of the individual and collective human behavior. In this context, it is surprising that our optimization dynamics considerations provides a believable explanation for the existence of human organizations which, although not optimal, persist. Works in order to investigate the influence of relevant individual differences onto the optimized hierarchy, are in progress. Kuramoto's models are suitable for this kind of study because they are easily adaptable to mimic human features like flexibility, tenacity or even leadership capacity.

SUPPLEMENTARY MATERIAL A:
IN WHICH CASES DOES THE OSCILLATOR DESCRIPTION APPLY?

It seems quite obvious that the success of a joint action requires the coordination of each involved agents. What is far from being straightforward is the possibility, for certain joint action, to reduce the description the action of each agent as a phase variable of an oscillator. We now discuss this point, analyze the assumptions we use and give a few examples to illustrate.

a) single oscillator

Whatever the joint action under consideration, there is a list of actions which can or have to be done by each agent, with a pointer indicating the current action in progress. In the straighforward example of a single dancer on a dance floor, this list consists of all the possible dancing steps. In this space of states, the succession of the dancing steps draws a confined trajectory. In the simplest cases, this trajectory is periodic (waltz, madison, twist), but quasiperiodic, chaotic and even self transversely crossing trajectories can be observed.

We will not consider such complex trajectories and will limit ourselves to situation where the trajectory in the space of all possible actions of a single agent is periodic. This is a very strong restriction, but it does correspond to numerous situations. First to come to mind: walkers, dancers, rowers, protestors chanting a slogant, applause, workers in an assembly line. Musicians, at the level of the sequence of notes, do not follows a periodic trajectory, but they strictly respect the regular succession of stressed and unstressed beats, the so-called tempo or rhythm.

The description of an agent's action as the phase variable of an oscillator seems less and less straigthforward when considering joint actions which are more and more sophisticated. Why? As an example consider the problem of writing a very large computer program. For each procedure he has to deal with, the software ingenior has to draft the specifications, define the inputs and outputs, make some bibliographic research and either select or invent a suitable algorithm, generate the computer code, compile link and debug it. He has also to coordinate his action with contributors who are dealing with procedures which are either client or server. Indisputably at this level of description, the ordered sequence of actions is repeated identically for each new procedure. Nevertheless, we are very reluctant to assume a periodic trajectory because we know, by experience, that the execution of such a complex task can be stopped by many unpredictable obstacles. What may prove decisive for the project progress and success is the exceptional brilliant idea that may germinate (or not!) in the brain of a single contributor. By their very nature, outstanding contributions don't occur periodically or even regularly. Therefore in the present study, we limit ourselves to situations where each agent is assumed to be competent enough, full of goodwill, and able to come to grips with the task he was assigned to in the allotted time. We consider situations where nothing extraordinary is expected from any contributor, neither decision nor choice, but only routine and know-how built up on the repetition of the same actions.

b) several oscillators

Moving his legs one after the other, a single walker performs a regular and periodic sequence of actions. This natural rocking frequency may slightly changes from one agent to the other, depending on the length of his legs, his weight or his size. Therefore when a group of walkers work together to carry a heavy mass, they have to coordinate themself and to adapt their rocking frequency to a common one. They have also to lock their phase in order to prevent parasitic movements of the heavy mass (pitch and rolling). Despite this coordination process, the trajectory of each agent in the space of states, is still almost periodic and only slightly disturbed compared to the situation where he is alone. It is worth noting that it is not always the case. For example in a assembly line, some workers may decide to occupy a position rather than an other depending on the delay, hence switching from one trajectory to another. Here, we will not investigate such situations and limit ourselves to cases where the collective behavior only slightly impact the individual one.

SUPPLEMENTARY MATERIAL B:
DYNAMICAL VERSUS STRUCTURAL STABILITY

A eight rowing team is a striking example of coordinated periodic behaviors, where each rower, except the first one, must absolutly synchronize his movements with the others on pain of being hit by his oar and stopping the boat. A pertinent and usefull remark is that the desired synchronized state has to resist to small perturbations like a momentary lapse of attention or the crossing of the wake of a boat. In dynamical systems theory, this ability to withstand perturbations is called dynamical stability and is characterized by the exponential decay rate of the deviations with time, the shorter the resilient time the higher the stability. When there exist several decay rates (depending on the type of perturbations), the smallest and therefore most critical is the relevant one.

Now assume that an eight rower team has been set up and successfuly tested in the sense that it is able to sustain a high row rate of 45 oar strokes per minute for the whole race. A new organization of the crew is then tested, where the first rower exchanges his place in the boat with the sixth. This new organization can be seen as a modification of the first one, the exchange which has been carried out as a perturbation, and the question of whether the new organization will be able to successfully sustain the same high row rate as a stability question. This type of stability is called structural stability and is associated with the dynamical process of improvement of the crew organization. Hence there exist two different characteristic time scales, a short and a long ones. The former is associated with the response of a given organization to a perturbation (lapse of attention, wake of a boat), the latter with the slow evolution of the crew organization to improve its performance.

SUPPLEMENTARY MATERIAL C:
IMPROVEMENT OF THE JOINT ACTION PERFORMANCE

Consider again the carrying of a heavy mass by a group of people. Even for this simple joint action, the improvement of the crew's performance may be understood in several distinct ways, most often conflicting: increase of the maximum mass which can be carried, increase of the travelled distance, decrease of the number of carriers, of their fatigue, or of the risk of an accident. Hence, not only the optimum network of collaboration is expected to depend on the nature of the joint actions we consider, but also on the kind of performance we want to improve.

In the present study, we will consider that the improvement of the performance has to be understood as an increase of the sole dynamical stability of the organization, i.e. its ability to withstand small perturbations and to recover from disturbances. This restrictive choice is motivated by two reasons: First, whatever the type of performance we want to improve, dynamical stability is always additionnaly (and most of the time implicitly) required. After all, what would be the usefulness a assembly line designed for a very high flux, but which never works because of incessant breakdowns? Second, dynamical stability is an universal quantitative criterion which allows a comparison between a wild variety of organizations.

References

[1] W. Allee, Social dominance and subordination among vertebrates, Biol. Symp. 8, p139 (1942).

[2] A. Guhl, Social inertia and social stability in chickens, Animal Behavior 16 (2-3), p219 (1968).

[3] E. Wilson, The insect Societies, ISBN 9780674454903, Harvard University Press (1971).

[4] I. Chase, Social process and hierarchy formation in small groups of laying hens : a comparative perspective, American Sociological Review 45 (6), p905 (1980).

[5] C. Castellano, S. Fortunato and V. Loreto, Statistical physics of social dynamics, Review of Modern Physics 81, p591 (2009).

[6] E. Bonabeau, G. Theraulaz and J. Deneubourg, Phase diagram of a model of self-organizing hierarchies, Physica A 217, p373 (2006).

[7] D. Stauffer, Phase transition in hierarchy model of Bonabeau, Int. J. Mod. Phys. C 14, p237 (2003).

[8] K. Malarz, D. Stauffer and K. Kulakowski, Bonabeau model on a fully connected graph, Eur. Phys. J. B 50, p195 (2006).

[9] L. Gallos, Self-organizing social hierarchies on scale-free networks, Int. J. Mod. Phys. C 16, p1329 (2005).

[10] T. Odagaki and M. Tsujiguchi, Self-organizing social hierarchies in a timid society, Physica A 367, p435 (2006).

[11 M. Tsujiguchi and T. Odagaki, Self-organizing social hierarchies and villages in a challenging society, Physica A 375, p317 (2007).

[12] H. G. Landau, On dominance relations and the structure of animal societies: I. Effect of inherent characteristics, Bull. Math. Biophys. 13, p1 (1951).

[13] H. G. Landau, On dominance relations and the structure of animal societies: II. Some effects of possible social factors, Bull. Math. Biophys. 13, p245 (1951).

[14] I.D. Chase, Dynamics of Hierarchy Formation: the Sequential Development of Dominance Relationships, Behaviour 80, p218 (1982).

[15] R. C. Francis, On the Relationship between Aggression and Social Dominance, Ethology 78, p223 (1988).

[16] I. D. Chase, C. Tovey, D. Spangler-Martin and M. Manfredonia, Individual differences versus social dynamics in the formation of animal dominance hierarchies, PNAS 99, p5744 (2002).

[17] M.G. Zimmermann, V.M. Eguïluz and M. San Miguel and A. Spadaro, Cooperation in an Adaptative Network in Application of Simulations to Social Sciences. Eds. G. Ballot and G. Weisbuch , Hermes Science Publications, p283 (2000).

[18] M. G. Zimmermann, V. M Eguïluz and M. San Miguel, Cooperation, adaptation and the emergence of leadership, in Economics with heterogeneous interacting agents, Springer Berlin Heidelberg, p73 (2001).

[19] J.A. Acebron, L.L. Bonilla, C.J.P. Vicente, F. Ritort and R. Spigler, The Kuramoto model: A simple paradigm for synchronization phenomena, Rev. Mod. Phys. 77, p137 (2005).

[20] J. Tōrōk and al, Opinions, Conflicts, and Consensus: Modeling Social Dynamics in a Collaborative Environment, Phys. Rev. Lett. 110, p088701 (2013).

[21] Y. Kuramoto in International Symposium on Mathematical Problems in Theoretical Physics, edited by H. Araki, Lecture Notes in Physics 30, Springer, New York, p420 (1975).

[22] For example, an other possibility could have been : $mass(A) = \max_{j,i} |A_{j,i}|$. From a mathematical point of view, this current definition is as good as the previous one (eq. 2), although the interpretation is not the same at all. Here the constraint applies individually on each participant while it applies globally on all the participants in eq. 2.

[23] T. Nishikawa and A. E. Motter, Network synchronization landscape reveals compensatory structures, quantization, and the positive effect of negative interactions, PNAS 107, p10342 (2010).

[24] B. Ravoori and al., Robustness of Optimal Synchronization in Real Networks, Phys. Rev. Lett. 107, p034102 (2011).

[25] Duan Zhi-Sheng and Chen Guan-Rong, Does the eigenratio λ_2/λ_N represent the synchronizability of a complex network?, Chinese Phys. B 21, p080505 (2012).

[26] A. Schwenk, Almost All Trees Are Cospectral in New Directions in the Theory of Graphs, edited by F. Harary, Academic Press, p275 (1973).

[27] E.A. Variano, J.H. McCoy and H. Lipson, Networks, Dynamics, and Modularity, Phys. Rev. Lett. 92, p188701 (2004).

[28] J. Touboul, The hipster effect: When anticonformists all look the same, arXiv:1410.8001.