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Mathematics has allowed physical theory to predict and derive testable consequences from claims about the nature of the physical world, and to verify from evidence whether these are correct. Mathematics has a similar impact on cultural theory. While most the mathematics summarized here has been published in other journals, the composite effect has not been summarized. Part 1 describes how to represent the kinship system of human cultures. Denotation of kinship may be different for distinct cultures, but the number of abstract representations is much more limited than the much larger number of human cultures. Kinship likely affects how marriage is decided, therefore is an essential part of describing how the culture survives. Viable (surviving) cultural systems (which we call "histories") are described by mathematical groups. In Part 2, the order of each kinship group leads to verifiable predictions of the demography and changes in the demography of each the history. It does so by the mathematically required use of the Stirling Number of the Second Kind (SNSK); we refer to published examples. Using SNSK also means that population statistics must assign unique individuals to indistinguishable roles in a common culture – a fact required by the mathematics, not an assumption of the theory. But the required statistics to test this are therefore very different from what is often claimed for "testing" theory. Part 3 then discusses distribution of a species cognitive systems, in humans. Part 4 discusses that while mathematical anthropology is not a "quantum" theory, the mathematical topic called "quantum logic" uses both full and partial algebras, thus is useful in describing the theory, and its limits. Part 5 summarizes the previous topics. The implication of the SNSK requirement is for all species, not simply humans.

1. KINSHIP AND MARRIAGE

We begin with our past. The earliest mathematical anthropological paper is from 1882 [1] which modeled strings of symbols to represent semantic concepts, such as "MBD" (mothers brothers daughter) for an instance of part of the English term "first cousin". Ethnographer Ruheman [2] in 1945 created the first description of kinship and marriage rules which identified objects that are clearly groups, though she does not use the mathematical notions to describe them. She denoted each cultural system by placing names of kinship relations onto finite sets of offspring, used diagrams to show how a terminology relates to sets of parents, and represented each cultural system by separate "generations". [2] page 543 said:

> "… The system must be self-contained and consistent; must provide a sufficient number of descent lines
>
> (a) to enable every member to select a spouse from his own generation;
>
> (b) to allow space for every distinct relationship term; and no term may appear more than twice … in any one generation.

"… The system must be able to reproduce itself after a number of generations. …"
Ruheman says her method had these advantages ([2] page 576):

"… It is independent of any special principles, which might at best be controversial, and proceeds simply from a systematic arrangement of the recorded facts;

"… It enables the dominating features of even a very complex system of kinship terms to be brought into full view in a single picture;

"… It makes possible the comparison and classification of widely divergent kinship systems, and provides a useful check on the completeness of our information."

Ruheman's methods do not therefore discuss many of the issues often treated in non-mathematical analysis of cultural systems. Avoiding non-mathematical descriptions is one of her purposes.

Depicting Kinship Using Groups

Ruheman illustrations use " = " for the "marriage" between two individuals; two sets of parallel but opposite arrows as "brothers and/or sisters"; and placed objects into columns labelled m and f for males and females. In Figure 1 from [2] page 547, she used labels A and B for moiety names:

FIGURE 1: EXAMPLE FROM RUHEMAN'S STUDY OF AUSTRALIAN KINSHIP

	f_1		m_1		f_2		m_2	
→	A	=	B	⇄	B	=	A	←
=	B	⇄	B	=	A	⇄	A	=

Descents of a male of A are also called A; descendants of a male of B are also called B; the top layer is one generation of two sets of moieties, with two marriages, which reproduces in one generation another set of A and B moieties. The arrows or = signs on each end of each line connect to the persons on the other end of each line. This system thus has two sets of marriages in each generation; each A always marries a B; and it has what we later call a "structural number" s = 2. Ruheman says that a representation must have a "sufficient number of descent lines" per generation to allow each individual in the structure to have at least one marriage partner; thus the "minimal structure" (that meets to the minimum as implied by Ruheman) is a description of the rules, not a definition of the necessary configuration of specific arrangements of each culture defined. But the "size" s of that minimal structure is very important to the results discussed later here.

The most referenced "first" version using group mathematics for kinship was by Andre Weil [3] as an appendix to Claude Levi-Strauss' 1947 *The elementary structures of kinship* [4]. Weil's initiative was initially expanded by [5, 6, 7, 8]. Use of the related Cayley groups was described in [9]. While groups require an associative operation, the first work showing that kinship rules are non-associative algebras was [10]; but a non-associative algebra, which includes most languages for example, can have an associative sub-set (the kinship rules for example), which are allowed by [10] and required by [11]. [12] presented the first paper which derives the group math in anthropology entirely from purely mathematical assumptions. [13, 14] elaborated many group-theoretical examples of kinship systems. [15] reviewed Weil's original work and its subsequent history.

[16] extended Weil's work by using semigroups and homomorphisms among semigroups, as well as groups. [17] expanded on the role of structural relations and graphs on cultural structures. Network analysis used semigroups and semigroup isomorphisms, including extending to relations other than kinship concepts: see [18, 19, 20, 21, 22, 23, 24]. What is, to this author, interesting is that the extension of algebras onto *any* cultural "networks" has led many anthropologists to describe many empirical networks using various devices for representations. But those in which the networks are also specifically groups on kinship relations have also allowed additional mathematical discoveries, including those on sizes of populations and advances in cognition as described later in this article.

Since each culture may have its own kinship descriptions, describing each one is a major area of work. Extension of Weil's ideas include [25, 26, 27, 28, 29, 30]. [31] showed how to create ethnographic kinship data collection using integrated mathematical analysis. In 2018 a series of eight articles greatly expanded the depth of analysis of kinship [32, 33, 34, 35, 36, 37, 38, 39] -- due to the principal author, we call that set the "Read articles"; readers should see those articles also for their added citations. Many of the Read articles are also seeking to relate the kinship groups directly to other concepts of cultural order; [10] sought the same results. Based on the Read articles and the other history presented, Read [33] page 54 cited the necessity of mathematical groups:

> "… Because the core structure for classificatory terminologies has inverses and not just reciprocity between ascending and descending kin terms, the core structure is a group; that is, an algebra with an (associative) binary product, an identity element, and an inverse for each element in the algebra."

Read is thus also describing the empirical facts that make Ruheman's concepts, necessary. Just as in physics, therefore mathematical groups have become one of the more significant aspects of mathematical anthropology.

Additional Examples in Mathematical Anthropology

Nearly all of the works cited above assumed that each description was of one (presumably internally coherent) culture at a particular time. [40] pg. 484 noted "each series of successive phenomena [is] not just in space but in time". [41] treated cultures based on just one description but with longer term changes, using the logistic equation. As archeologists, Renfrew and Cooke [42] advocated longer term descriptions of culture than do cultural anthropologists, including traditional demography, descriptions of matrix and other means of describing cultural form and change, aspects of genetics and multipart evolution, time dispersal of artifacts, analysis of centuries or longer term cultural descriptions, and many other topics; they described consistent patterns in cultural descriptions, without using the math of group theory. Other work followed methods of other sciences, including mathematical micro-economics, game theory, statistical analysis, network theory, and others: especially [43, 44] followed Hirshleifer's [45] demonstration that micro-economics describes certain long term cultural evolutionary changes; also [46, 47, 48, 49, 50, 51, 52, 53] and other collections; and specific works such as [54] on iconography, [55] on proofs in Swahili, and the notion "ethnomathematics" such as [56] which treats Weil's ideas as but one chapter.

2. CULTURE AND POPULATION MEASUREMENTS

Ballonoff [11, 47, 57, 58, 59] follows the use of longer time periods of evolution, and the use of mathematical group theory, but also derives results based on the number of *histories* used in each of those time periods; we describe the "history" notion more fully below. While most mathematical anthropology articles cited above treat each culture as having one history, Ballonoff allows that more than one history could be present in each generation of a single cultural system, and discusses the consequences of that. His work shows that the operation of a specific history has a minimal description (as also implied by Ruheman). Knowing the order of each system kinship group and whether there are changes in the relative proportions of histories in one society per generation, therefore predicts changes in the demography of any society following one or a mixture of specific histories.

To explore this [60] defined:

$$\begin{array}{ccc} & D & \\ G_t & \rightarrow & G_{t+1} \\ \downarrow\mu & & \uparrow\pi \\ M_t & \rightarrow & B_{t+1} \\ & d^{-1} & \end{array}$$

where:

D	summarizes biological population operators for genetics and demographics of the the population at the indexed time t,
M_t	the sets of married couples at time t, with $M_t \subseteq G_t$
B_{t+1}	a partition of G_{t+1} into sets of persons with the same parents, with $B_{t+1} \subseteq G_{t+1}$
μ	a surjection corresponding to assignments of M_t,
π	a partition of G_{t+1} showing kin groups of a population within a generation t+1, as assigned by the cultural rules of marriage
d^{-1}	a surjection corresponding to descent
d	an injection corresponding to ancestor

so that the d^{-1} surjection maps the progress of population change following sets of descendants in generation t + 1 onto the sets of parents in generation t.

Cultural Rules and Population-Size Changes

Many things affect population size; this article is not an attempt to identify nor describe most of those. For example, the formulas in D are results from biological processes. The demographic idea of the Leslie matrix [61, 62] finds a growth rate and stable age distribution as eigenvalues and eigenvectors of a matrix of age-structured mortality and fertility or fecundity rates; and reflects the approach of Lotka [63] on the normal demographic discussion of the growth of populations, analogous to "capital" growth in economics. Neither the Leslie matrix nor the Lotka model treats the cultural system, and thus do not treat consequences on population change due to changes in the cultural system. (Since we are studying cultural rules, not biological ones, we assume here the sex ratio is half male; those factors and many other details are studied in more depth in biological work such as by Lotka, Leslie, and many others.)

Cultural representations are the results of the "cultural rules", which we call the *history* = $\alpha = \{\mu, \pi, d^{-1}\}$. Given a history α, when we find the mathematical kinship group which is the smallest size and that repeats itself in the shortest cycle of generations (as implied by Ruheman), we call the number of marriages in that group per generation as the *structural number* of α and denote it by s. Thus the structural number relates to the definitions of the kinship, and ist not a description of the total society of some empirical example. The d^{-1} mapping of descent is a surjection (each set of the descendants in generation t+1 are defined as coming from a particular set of parents in generation t). Therefore theorems require that as a surjection, forecasting populations statistics of cultural systems requires using the Stirling Number of the Second Kind (SNSK) [see for example citations 64, 65 page 178, 66].

We first discuss how this affects the statistics of a society. Let s be the structural number of history α; let n_s = *average family siz*e of a system with structural number s; define the proportion of (socially ascribed) *reproducing females* as p_s; and let $p_s = 2/n_s$. The specific values of p_s, and n_s by structural number s are described in the Appendix of [11], implementing [57, 58], as fixed numbers, given s. Note that n_s is always ≥ 2, so n*p = 2, or

$$1. \quad \tfrac{1}{2}(n_s{*}\, p_s) = 1.$$

Because the exponential form $e^{r(t)} = 1$ then we can also derive a general "equity growth" equation for any system with structural number s:

$$2. \quad e^{r(t)} = \tfrac{1}{2}\, n{*}p.$$

The equity growth r(t) is distinct from the notion of capital growth idea per Lotka, which we here call R(t); below we derive an estimate of capital growth R(t) from the equity growth r(t).

When we use specifically the values n_s, p_s given a history of structural number s (see also [11, 57]) then since $n_s p_s = 2$, Equation 2 results in r(t) = 0. Therefore r(t) = 0 shows that the values of n_s and p_s allow a culture with history α and structural number s to have neither growth nor decline. As structural number s increases (above s = 3), then from [11, 57] values of n_s increase and p_s decrease, to maintain cultural stability with zero change r(t) = 0. As the cultural systems gets "more complex" (as the structural number s increases) then the average family size n_s per house-hold also increases; this is similar to the notion that as general systems become more complex, their size must increase to maintain the same level of reliability – which here is when r(t) = 0.

Let the population at time t use some set of histories α, … β. Then the proportion of G_t using α is $v_{\alpha t}$, the proportion of G_t using history β is $v_{\beta t}$, etc. We require that $0 \leq v_{\alpha t}$,, …, $v_{\beta t} \leq 1$ and $\Sigma v_{\alpha t}$, = 1 over all histories allowed in time period t. Then the average $n(t) = \Sigma v_{\alpha t} n_s$ over all histories allowed at time t; the average $p(t) = \Sigma v_{\alpha t} p_s$ over all histories allowed at time t; and we get an equation that allows prediction at time t of the time-dependent average population measures for n(t), p(t) and r(t) as the percentages of histories α, …, β change:

$$3. \quad e^{r(t)} = \tfrac{1}{2}\, n(t)*p(t)$$

From [11, 57] r(t) = 0 whenever all rules have the same structural number (or all have only s = 2 or 3); but $r(t) \neq 0$ whenever there is a change in cultural dynamics by allowing rules with more than one structural number (and at least one of structural number > 3); that is, this causes population growth (or decline). Otherwise stated, whenever more than one α is allowed at t (and at least one of structural number > 3) that have $0 < v_{\alpha t} < 1$ then there are at least two such histories, therefore $r(t) \neq 0$, and therefore the population statistics n(t), p(t) and thus r(t) of the system are changing. This result empirically is well known by anthropologists: as cultures change, population statistics can also change. It is *predicted* by cultural theory.

To relate r(t) to the demographic notion R(t) we applied mathematics developed for analysis of financial leverage (see [48] Chapters 3 and 4, [57] especially its Equation 17, and [58]) and found that:

$$4. \quad R(t) = 2\, p(t)r(t) / [p(t)^2 + 2r(t)].$$

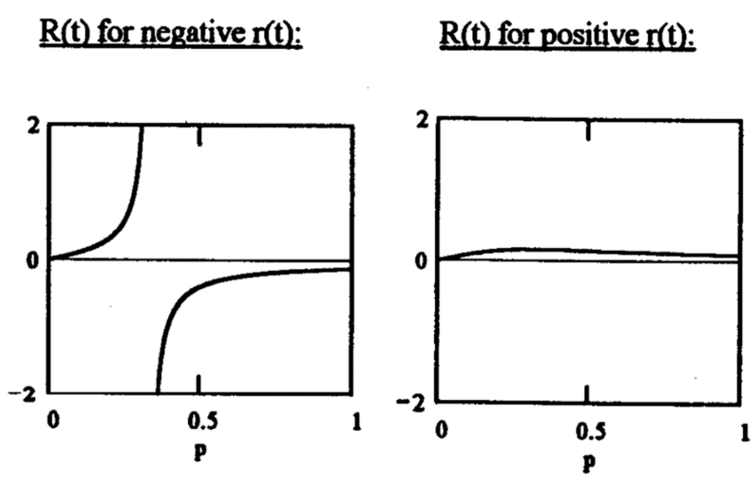
Equation 4 is called the *amplification equation* since it increases the usually much smaller value of r(t) into a larger value R(t). It can also be called "amplification" of local survivability, or, by analogy to borrowing of capital in the world of finance, "leverage".

Examples of Applications of the Theory.

These equations allow us to test our cultural theory on actual data. They allow predictions of population measures (and of changes in them) when only the cultural rules are known. Thus [46] retrodicted the population change experienced in Western Europe for the last 1000 years (using data in [67] page 79) based on knowledge of the change in cultural structure alone. [57] permitted computation of the possible ranges of village sizes for a Kashmiri Pandit population (from [68]) based on knowledge only of the cultural structure, and allowed to predict ranges of values of the Moenkopi Hopi village sizes (from [69]) based also only on knowledge of the cultural structure. In [70] the separation of an Apache village into two new villages was predicted by upper and lower bounds of allowed statistics computed based solely on the knowledge of cultural structure. [43] predicted average U.S. population statistics for stability from knowledge of cultural rules of marriage. The theory has also been shown capable of interpretation of demographic characteristics of India at the local scale ([57] page 112). Also see [11, 44, 71, 72, 73].

The amplification equation, Equation 4, also contains a singularity, when in the denominator $p(t)^2 = -2r(t)$ -- see Figure 2, whose values come from [74, which includes Figure 2 derivation]. A system with negative r(t), on the left-hand image of Figure 2 (the figure under "R(t) for negative r(t)"), will likely itself go extinct – negative r(t) itself predicts that. But if some species did exist on the left side of Figure 2, it would likely move toward the right of that image, and will therefor also quickly become extinct as it reaches or exceeds the singularity. Therefore, Figure 2 implies that viable species are likely on the right side of Figure 2 (under "R(t) for positive r(t)") - that is, will require that viable species create a positive r(t); except when p(t) = 0 (which is anyway extinction), or when p(t) = 1 (and therefore we have only one history in the population at t). This also implies that while we measure n_s and p_s for the "stable state" of a system with structural number s by finding $e^{r(t)} = 1$ with r(t) = 0, for a system to survive longer term it should also expect to find some additional growth than just the n_s and p_s values -- if either or both empirical statistics of a cultural system exceed the n_s and p_s values, this will occur.

FIGURE 2: COMPARISON OF GROWTH RISKS R(t) IN VERTICAL SCALE DUE TO THE SINGULARITY IN THE AMPLIFICATION EQUATION AS p(t) MOVES FROM p(t) = 0 TO p(t) = 1



The "Uniqueness of Individuals" Requirement

Using SNSK is often described in math books ([65] page 178 for example) as placing unique individuals (each individual in the population being considered is unique) onto classifications of cells, each of which cell has identical description to all other cells. The cells here are the marriages as defined by the history. That description is also what one wants for describing "roles" in a culture. Anthropologists are thus required to use SNSK to allocate unique individuals into marriage units which have specifically defined roles in the society. The required use of the SNSK assumptions, unique individuals onto similar roles, is thus exactly what should be required for a theory certainly of human cultures. But nothing in our previous discussion requires this result; it is placed on us entirely by the theorems of mathematics.

Understanding of the uniqueness requirement thus needs a much deeper discussion, which we briefly begin here. One answer is that each individual human has in their RNA/DNA a particular individual combination of genetic codes. There are some unique exceptions, such as presence of twins (etc.), but those are small portion of populations (and also then get unique experiences, so also become unique from other means). Biologists who have considered diversity also question whether even "identical twins" have identical inheritance due to recombination -- see [75] page 11, published in 1956. Geneticists, in a "new" discovery in 2010 also say that the human genetic diversity is more caused by the effects of recombination factors in aligning DNA/RNA codes [76]. Further, the argument that inheritance requires recognition of individual distinction applies to any organisms that are related to parents by surjections; that is, almost any organisms. Medawar [77] page 185, in his classic *The Uniqueness of the Individual*, concludes "So far from being his [i.e., humanity] higher or nobler qualities, his individuality shows man nearer kin to mice and goldfish than to the angels; it is not his individuality but only his awareness of it that sets man apart". We do not comment if other animals might also be aware of their uniqueness. But geneticists now say that a similar source of wide genetic diversity is due to recombination (not simply to diversity of DNA/RNA), is also the case in yeasts [78].

Note that the methods often taught as "the statistics for testing theory" do *not* use the SNSK, and do not require recognition of uniqueness of individuals. They are generally based on the methods of classical physics. In general those statistics require the opposite assumptions as for SNSK: that we are assigning identical "individuals" into unique positions – for example identical atoms of air into unique positions in a room. Much of that statistical orientation has not led to long term lasting results in applications on humans. The American Statistical Association [79] has begun to address this; but see also Baily [80] - as for the Read papers, in [80] the citations should be looked on as added resources. Considering that the questioning of traditional methods for testing theories was already included in [77] in 1956, it is remarkable that it was only in 2016, 60 years later, that the statistical association took seriously the long-standing inability of these methods in many applications.

Thus, understanding what is meant by "uniqueness" requires a much deeper discussion than our current summary. But the fact that inheritance is a surjection, that therefore implies the use of the SNSK, which thus says that all individuals (of any species) are considered "unique", has very important implications for cultural theory, and indeed for evolution itself.

3. EVOLUTION OF COGNITION

Much of current anthropology seeks to be a cognitive science; we do not here review that literature. However [81] shows there is an isomorphism between the structure of RNA/DNA and a specific form of kinship. Haldane in [82] and many other citations models biological inheritance with methods similar to our formula for r(t). Haldane measures n(t) of each genetic code by the numbers of offspring which survive, and thus transmit the biological inheritance from their parents. For a cultural history α, the assignment of offspring to each family is done by how the culture "treats" the offspring; for example, cultures may allow adoption of offspring to be assigned to a specific set of parents; assignment is thus not necessarily genetic, but in general assignment is predominantly genetic. Haldane therefore models evolutionary biology consequences on populations statistic by similar methods to those used here.

Human and other species with more elaborate neurological systems tend to have much higher p(t) values, and very low n(t) values. (They are therefore within the positive r(t) section of Figure 2.) They empirically typically have much longer times for individuals to "mature" than do most other species. They are much more likely to have inter-individual communications based on neurological devices, not only biochemical means. Their "culture" can be more closely related to their neurology, and may change much more quickly by learning. They often have much longer periods in which their offspring become "adults" and thus to have more time to learn the things that species with a higher neurological system can offer. While essentially all species have individuals that use internal biochemical communication, not all have well developed secondary (nervous) systems. Humans do, as do most other mammals, birds, reptiles, and fish. Once a secondary neurological system exists in a species, its symbolic capabilities facilitate better and more rapidly changing inter-individual communication; cultural change can happen within the life of single individual.

Now consider a social insect. In general each system per generation has one female parent and very many offspring; social insects thus have very low values of p(t) and very high n(t). For inter-individual communications social insects more likely use biological mediation (genetically determined communication, or pheromonal means of communication, also determined by genetics). As social insect cultural systems are thus predominantly genetically determined, therefore they have much more stable "cultural" systems — witnessed that diverse species of social insects have existed for very long periods in apparently very similar condition to those social insects which exist today. In general social insects are distinct species for each environment; slowly evolving DNA may account for this.

Also note: a social insect species could be on the right-hand side, positive r(t) section, of Figure 2, near the left part of that diagram. But it could also have a negative r(t) and be on the left-hand side of that illustration; generally that would quickly lead that species to the singularity and extinction. But if such species of social insects were on the negative r(t) of Figure 2 and could somehow also retain their very low negative r(t) – for example, by keeping the size of negative r(t) sufficiently small so that it stays below the value of $p(t)^2$ in the left side of Figure 2 – and as social insects have very high p(t), this configuration may thus yield a positive R(t). That is, they could also retain a system with long term stability of their culture, due to the slow changes of genetically determined cultural systems. The singularity of the amplification equation may also explain why we have both long-term stability of species, and diverse species by environment of, social insects.

Modern human environments (and of other animals with more developed neurological systems with greater awareness of properties of symbols) thus induces natural selection that favors individuals with increased capabilities using the symbolic communications and control systems. Thus the human world (and likely that of birds and many other species) is of increasingly complex symbolic systems - especially using not just language but "entertainment" or other forms of visual or symbolic demonstration, and thus including mathematics, politics, law, and many other abstract systems – which are therefore simply an extension of the same processes of natural evolution. We extend this discussion in Part 5 below.

This conclusion is also different from what the group often called socio-biologists claim. Lumsden and Wilson [83] have analyzed whether a culture can "have a life of its own". They have analyzed if cultural kinship rules can avoid "incest" in small populations; in fact, they can do that. But [11, 57] have an important distinction: they demonstrate that if a species has a population that uses a history with a structural number, that the predicted demography (specifically, the p(t) and n(t) values) are dependent on the structural numbers of the histories alone. The structural numbers are in general not related to the absolute size of the empirical population. Humans can evolve by advancing neurological abilities simply because humans have the ability to create conceptual (cognitive) means of organizing. The human ability to create for example kinship systems or histories that are also groups indicates evidence of the underling human neurology. This is also not a claim that specific histories or their structural numbers themselves cause the changes in neurological abilities. Recent studies [84, 85] on the role of kinship have also reached the same conclusion; and see also Part 5 below. Thus socio-biologists hypothesizing a culture acting at the of the size of the minimal system might indeed reduce "incest" is mathematically correct, but is unrelated to the role of kinship in human cultural systems. A reader can decide if the other claims of [83] may be true, but they are independent of what cultural theory as discussed here implies.

4. RELATIONS TO QUANTUM LOGIC

Ballonoff has published a series of papers [11, 59, 86, 87, 88] based on inspiration from [89] using methods of quantum logic in presenting cultural theory mathematically. While the methods have been used in constructing parts of quantum theory, the separate methods alone are not quantum theory. Their advantage to cultural theory is that quantum logics have many devices that can treat systems in which the basic algebras are not full. In empirical studies of culture, there are probably less than 20 distinct forms of how kinship systems may be created (though there may be thousands of cultures with distinct languages in which they are expressed), and most of those systems have structural numbers at or well under s = 16. If we look at combinations of cultures and seek to create a lattice of all structural numbers in the lattice, the product of all structural numbers available can create a lattice whose top-group order may therefore be significantly above 16. But there may be no such empirical systems. Another way to state this is that we may have sets of possible histories, not all of which can be allowed to find each other from within a common set. That in turn may also create changes among histories in which some ranges of values of n(t) and of p(t) -- hence of r(t) and R(t) -- that are also not allowed. The homotopy of these systems may therefore tell us a lot on which kinds of cultural systems might evolve into each other or in which sequences, and which if any might not evolve; see [11] for more discussion. The possible limits to which sequences of histories might be allowed or inhibited to co-exist needs further work, both mathematically and empirically.

Many ethnographies discuss the cultural systems as if there is only one history present in that population. In many papers cited here, therefore their discussion does not talk of "probabilities" that a culture is in some specific history; they assume a specific history, and thus no probabilities are needed. Applications of the same history at each instance of time also means that there is no change in demographics of the population: if the same structural number repeats, then the predictions of n(t) and p(t) do not change; they remain at n_s and p_s. [11, 57] demonstrated that either cultural histories of a society follow one history, consistently, or may have more than one history possibly present at each instance of t. But in that case probabilities are needed - in [11] they use Pauli coefficients, as the method simplifies following specific paths. But if a culture changes between two different histories with different structural numbers (with at least one having s > 3, see [11, 57]) then $r(t) \neq 0$ and thus also $R(t) \neq 0$. That is, as noted earlier, cultural theory tells us that culture change (change in relative application $v_{\alpha t}$ for each α) causes changes in population statistics as well.

Similar mathematical forms occur in quantum logic. Quantum logic [90, 91, 92, 93] defined a "consistent histories" approach, in which the same history is consistently followed in each generation of a descent structure. This approach defines a *descent map* (called "descent" in quantum theory as well), which, like the descent map of cultural theory, is a partial order. But in physics, the physical object in each instance of t is a copy of the same object as the previous instance; only its momentum or location may have changed. In culture theory we do not have momentum and location, we have the values of n and p. In cultural theory the objects in each case is defined by a specific history $\alpha = \{\mu, \pi, d^{-1}\}$ which determine the n and p. And the objects at t+1 are descendants of the objects at t, they are *distinct* objects, not the same objects as in physical theory. In both examples, we have groups which recreate in each generation an object defined by those groups, but a repetition of the literal same object in physical theory is not the same as a distinct object in cultures or in other biological systems. See [11] for the similarity of the equations. Objects discussed in physics as non-commutative have also been found in physics to have "uncertainty". [11] instead found that when objects in culture theory are similarly non-commutative that those cultures instead have a change in their population statistics. Therefore, instead of "uncertainty" in physics, in cultural theory non-commutativity in these equations forecasts $r(t) \neq 0$ and thus also that generation growth (or decline) occurs. Thus, while we can use devices that are similar to those found in physics, our results may have very different consequences than the physical applications.

5. FURTHER CONCLUSIONS

Part 4 summarized of some methods of quantum logic that assisted presentation of the current theory. [11] presented parts that reflect similar formulas in physics, but have very different meanings. Similar results also occur when comparing cultural theory to parts of population genetics. Part 3 discussed that [83] seem to show that minimal systems also seem to avoid "incest", but as discussed in Part 3, avoidance of incest is an effect of the rules on small populations, but not the purpose of those rules. The formulas of [11] have the same maximum and minimum probabilities as those of [94] Chapters 1 and 3. But [94] relates to effects of measures of inbreeding, and in that sense relate to showing directions of biological evolution. The results of [11] do not show that if a culture changes from history α to history β that the change of population statistics as result are genetic evolution of the system. Instead, they simply show that the society by moving from α to β may change its cultural population statistics; the same population could in the next period change back from β to α, changing the statistics back to those of *a*. There is no (permanent) biological "evolutionary" change under cultural theory.

Part 3 noted that culture theory in predicting population statistics are "similar" to how Haldane measures genetic offspring per couple. But Haldane measures and uses only the specific genetic offspring of the parental couples. Culture theory predictions of n and p are "largely" but not entirely based on genetic offspring of each parental couple, but also involve a component of offspring from other parts of the population -- such as by adoption, a cultural device, not a genetic one. Thus in cultural theory predicted average family size n_t is the proportion at time t of the entire species population; its sub-parts by its formula are not due to changes in genetics. None the less, as discussed below, as the species is better able to use symbolic means, that influence will therefore affect the genetics of the entire population.

Part 2 showed that the requirement that the offspring generation in relation to its parental generation was a surjection. Being a surjection meant (due to theorems cited above) that the offspring population would be all "unique" individuals, and be assigned as offspring to "identical" cells of the previous generation. Part 1 discussed that the role of marriage in human society is describable in part by mathematical groups, on the parental generation, describing how the "uniqueness" occurs. Indeed the paring of the parental generation in population genetics is also represented as mathematical groups – in fact [81] notes that DNA/RNA mechanics are not simply representable by groups, but are also similar to the groups used for human marriage rules. And the offspring are all unique in any species using surjection (see again [76], which for example refers to yeasts, not humans). Thus genetic inheritance also fills the surjection requirement as predicted by mathematical theorems: it creates unique individuals and allows for identical means of reproduction.

Now compare to the literature which discusses "cultural evolution". Gintis [95] shows which that interpretation such as by [94], that the advances of human altruism relates to mainly to the claim that the human kinship systems leads to avoidance of inbreeding, is incorrect. In some cases the small size of a population means that the literal application cultural kinship rules in very small populations does indeed avoid incest; but Gintis [95 Chapters 9 and 10] shows that inheritance of the population is not by avoiding "incest", but evolves by the "social structure" embedded in the entire genome of the species. This therefore reinforces the result found two paragraphs above: as the species is better able to use symbolic means, as shown by the cognitive ability to use symbolic means in the conception of social structure, that influence will affect the genetics of the entire population. [95] thus both favors the results of Medawar [77, page 185] as also stated in Part 2: "So far from being his [i.e., humanity] higher or nobler qualities, his individuality shows man nearer kin to mice and goldfish than to the angels; it is not his individuality but only his awareness of it that sets man apart"; and the result restated in [95 page 196] that "All members of a species share more than 99 % of their genes, so why shouldn't altruism favour universal altruism?" (see Washburn [96, page 415]). (This discussion also implies that the literature claiming to explain cultural changes as examples of cultural mechanics mirroring genetic inheritance, is also at least in one major part, wrong. That discussion however belongs in a separate document.)

Mathematical anthropology is in main part basic group theory and results from simple methods on population stability. Yet we also get significant results. We can test for the internal consistency of the histories given the description ethnographers make; we can use them to predict future events, including changes in demography, in a system following a given history or set of histories in defined proportions; we can use them to compute boundary conditions for the continued existence of following particular histories in defined proportions; we can use them to reconstruct a past state from knowledge of events and observables, or past events and observables from knowledge of the proportions of the available histories. We can use them to understand why evolution may create very distinct forms of communication between individuals in a species. The mathematically required use of the SNSK and thus the required use of assuming each individual is unique, requires additional study for creating cultural theory – it implies very different sets of tests for understanding cultures than are used by basic (and often wrong) past methods of "testing" theories. We thus believe there has been significant progress in the creation of a mathematical theory of culture. Cultural theory, aided by mathematics, is a proven form of description and prediction for important parts of the natural world.

References

1 Macfarlane, A., "Analysis of Relationships of Consanguinity and Affinity" in Journal of the Royal Anthropological Society 1882 Vol 12 pages 46 – 63.

2 Ruheman, Barbara. "A method for analyzing classificatory relationship systems" in Southwestern Journal of Anthropology 1945 Vol. 1 pages 531-576

3 Weil, A. 1947 Appendix to Claude Levi-Strauss, 1947.

4 Claude Levi-Strauss 1947 *The elementary structures of kinship* (in English from Beacon Press, Boston 1969)

5 White, Harrison. *An Anatomy of Kinship*, Prentice Hall, New Jersey. 1963

6 Liu, Pin-Hsiung. "Formal Analysis of Prescriptive Marriage System: The Murngin Case", in *VIII Congr. Anthropolo. Eth. Sci., Vol II Ethnology*, 1968, pages 90 – 92.

7 Liu, Pin-hsiung, *Murngin: A mathematical solution*, Institute of Ethnology. Academia Sinica, 1970, Monograph ser. B, no. 2)

8 Courrege, Philippe. "Un Modele mathematique des structures elementairs de parente", in *L Homme* Volume 5 No 3 – 4, pages 248 – 290, translated to English by D. Read as pages 289 – 338 in Ballonoff, P. A. (ed.) Genetics and Social Structure, Dowden, Hutchinson and Ross, Stroudsburgh Pennsylvania (1974).

9 Gould, Sydney and D. Kronenfeld *New System for the Formal Analysis of Kinship* University Press of America. 2000

10 Greechie, Richard, and Martin Ottenheimerm "An Introduction to a Mathematical Approach to Kinship" pp. 63-85 in *Genealogical Mathematics* (P. Ballonoff', ed., Maison des Sciences de l'Homme, Paris, 1974

11 Ballonoff, P. "Paths of Cultural Systems". *Entropy*. 2018, 20(1), 1 - 13.

12 Harris, T. R., "From Relations to Groups: A Formal Treatment of the Underlying Logic of Some Group-Theoretical Models of Kinship" pp. 85 -109, in in *Genealogical Mathematics* (P. Ballonoff', ed., Maison des Sciences de l'Homme, Paris. 1974.

13 Gottscheiner, A. On some classes of kinship systems, I: Abelian systems. In M*athematical Anthropology and Cultural Theory*, 2008; Volume 2, pp. 1–10.

14 Gottscheiner, A. On some classes of kinship systems, II: Non-Abelian systems. In *Mathematical Anthropology and Cultural Theory*; 2008; Volume 2, pp. 1–10.

15 Rauff, James R. "The algebra of marriage: an episode in applied group theory" (*BSHM Bulletin: Journal of the British Society for the History of Mathematics*, 2016, 31:3 pp. 230-244)

16 Cargal, James pp. 157-168 "An Analysis of the Marriage Structure of the Murnguin Tribe of Australia" (1968 *Behavioral Sciences* 23)

17 White, Harrison and F. Lorraine "Structural Equivalence of Individuals in Social Networks" in *Journal of Mathematical Sociology*, 1971, Vol. 1, pp. 49-80

18 Boyd, J. pp. 169 -172 "Structural Similarity, Semigroups and Idempotents" in 1983 *Social Networks* 5

19 Boyd, J. pp. 137-155 "Block Structures of Automorphism Groups and Social Relations" in 1988 *Social Networks* 10

20 Boyd, J. *Social Semigroups* George Mason University Press, Fairfax VA, 1991

21 Boyd, J. "Relational homomorphisms" in 1993 *Social Networks* 14 pp. 163-186

22 White, Douglas and K. Reitz "Semigroup Homomorphisms on Networks of Relations" in 1983 *Social Networks* 5, pp. 193-234

23 Patterson, Philippa *Algebraic Models of Social Networks* Cambridge University Press, Cambridge. 1993

24 Douglas White, 1974 "Mathematical Anthropology" pp. 369-446 in J. J. Honigman, *Handbook of Social and Cultural Anthropology*, Rand McNally 1974

25 Tjon Sie Fat, Franklin, "Circulating connubium and transitive ranking: a second solution to Leachs problem", in *Bijdragen tot de Taal-, Land- en Volkenkunde* 139 (1983), no: 1, Leiden, 140-151

26 Tjon Sie Fat, Franklin, "Age Metrics and Twisted Cylinders: Predictions from a Structural Model", *American Ethnologist*, Vol. 10, No. 3 (Aug., 1983), pp. 585-604

27 Tjon Sie Fat, Franklin "On the Formal Analysis of 'Dravidian', 'Iroquois', and 'Generational' Varieties as Nearly-Associative Combinations" pp. 59-93 in Tjon Sie Fat F.E., Godelier M. editors, *Transformations of Kinship.* Washington DC and London: Smithsonian Institution Press. 1998

28 Tjon Sie Fat, Franklin, *Representing Kinship: Simple Models of Elementary Structures*, (Thesis) 1990 Leiden.

29 Tjon Sie Fat, Franklin, et al, "On Mathematics in Structural Theory" [and Comments and Replies], *Current Anthropology*, Vol. 21, No. 3, June, 1980, 386 – 391.

30 Tjon Sie Fat, Franklin, et al, "More Complex Formulae of Generalized Exchange [and Comments and Replies], *Current Anthropology*, Vol. 22, No. 4 (Aug., 1981), pp. 377-399

31 Read, Dwight and Murray Leaf, *Human Thought and Social Organization: Anthropology on a New Plane* (Lexington Press) 2012]

32 Read, Dwight, The Generative Logic of Crow-Omaha Terminologies: The Thonga-Ronga Kinship Terminology as a Case Study. *Mathematical Anthropology and Cultural Theory*, 2018 Vol. 12 No. 1, pp. 1 – 37.

33 Read, Dwight, "Reply to Comments on The Generative Logic of Crow-Omaha Terminologies: The Thonga-Ronga Kinship Terminology as a Case Study. Mathematical Anthropology and Cultural Theory, 2018 Vol. 12 No. 8, pp. 1 – 62.

34 Heady, Patrick, "Comment on D. Read 'Generative Crow-Omaha Terminologies'", *Mathematical Anthropology and Cultural Theory*, 2018 Vol. 12 No. 2, pp. 1 – 8.

35 Tjon Sie Fat, Franklin, "Comment on D , Read 'Generative Crow-Omaha Terminologies'" *Mathematical Anthropology and Cultural Theory*, Vol. 12, No. 3: 1 - 10

36 Hamberger, Klaus, "Comment on D. Read 'Generative Crow-Omaha Terminologies' ", *Mathematical Anthropology and Cultural Theory*, 2018 Vol. 12 No. 4, pp. 1 – 9.

37 Patrick, "Comment on D. Read 'Generative Crow-Omaha Terminologies' ", *Mathematical Anthropology and Cultural Theory*, 2018 Vol. 12 No. 5, pp. 1 – 4.

38 Trautmann, Thomas R. and Peter M.Whiteley "Comment on D. Read 'Generative Crow-Omaha Terminologies' ", *Mathematical Anthropology and Cultural Theory,* 2018 Vol. 12 No. 6, pp. 1 – 12.

39 de Almeda, Mauro Barbosa "Comment on D. Read 'Generative Crow-Omaha Terminologies'", *Mathematical Anthropology and Cultural Theory*, 2018 Vol. 12 No. 7, pp. 1 – 23.

40 Bulgaru, Valeriu "The theory on historical series formulated by A. D. Xenopol as a mathematical approach to history" pgs 482-491 in F. R. Hodson, D. G. Kendal and P. Tautu, 1970, *Mathematics in the Archeological and Historical Sciences*, University Press, Edinburgh
41 Hamblin, R. L., R. B. Jacobsen and J. L. L. Miller *A Mathematical Theory of Social Change* (Wiley, New York). 1973
42 Renfrew. Colin and Kenneth Cook (eds.) *Transformations: Mathematical Approaches to Culture Change* (Academic Press, London) 1977
43 Ballonoff, P. *Mathematical Foundations of Social Anthropology*, Mouton, Paris. 1975
44 Ballonoff, P. *Mathematical Theory of Culture,* Austrian Society for Cybernetic Studies Monograph No. 40. 1987
45 Hirshleifer, J. "Economics from a biological viewpoint" pp. 1-52, 1977 *Journal of Law and Economics*, 20(1).
46 White, Douglas pp. 5 – 28 "Ring Cohesion Theory in Marriage and Social Networks" *Math & Sci. hum./ Mathematics and Social Sciences* 2004, 42, no. 168:4
47 Hans Hoffman "Mathematical Anthropology" in *Annual Review of Anthropology* 1969, Vol. 2, pp.189-199).
48 Michael Burton, "Mathematical Anthropology" 1973, pp. 41-79 in *Biennial Review of Anthropology*, 6
49 Paul Kay, *Explorations in Mathematical Anthropology* from the 1966 meeting of the American Association for Advancement of Science. 1972
50 Mitchell, J. Clyde (ed.) *Numerical Techniques in Social Anthropology*, Institute for the Study of Human Issues, Philadelphia, 1972
51 de Muer, Gisele (ed.) *New Trends on Mathematical Anthropology* (Rutledge and Kegan Paul, London). 1986
52 de Meur, Gisele and Alain Gottcheiner. "Prescriptive Kinship Systems, Permutations, Groups And Graphs" in *Mathematical Anthropology and Cultural Theory*, Vol 1 No 1, November 2000, http://www.mathematicalanthropology.org
53 Trappl, R, (ed.) "Proceedings of the 16th European Meeting on Cybernetics and Systems Research", Austrian Society for Cybernetic Studies, pp. 367 – 440 in Trapple (ed) *Cybernetics and Systems Vol. 2*, 2002, Vienna
54 Rauff, J. "Sand Songs: The Formal Languages of Walpiri Iconography" (*Humanistic Mathematics Network Journal* #15, 1997 pp. 17- 27.
55 Rauff, James R "Prove it in Swahili: Language as an Axiomatic System", *Illinois Mathematics Teacher,* Spring 2008 pp, 1–5.
56 Ascher, M. Ethnomathematics, *A Multicultural View of Mathematical Ideas*, Chapman & Hall, New York. 1991
57 Ballonoff, P. "Mathematical Demography of Social Systems" in Trappl, R. (ed.) *Progress in Cybernetics and Systems Research* Vol X. 1982, pages 101 – 112, Hemisphere Publishing Company, Washington.
58 Ballonoff, P. "Mathematical Demography of Social Systems II", in Trappl, R. (ed.) *Cybernetics and Systems Research* Vol, XI, 1982, pages 555-560, North Holland Publishing Co.
59 Ballonoff, P. "Manuals of Cultural Systems". *Int. J. Theor. Phys.* 2014, 53, 3613–3627.

60 Duchamp, T., P Ballonoff, in Chapter 3, of P. Ballonoff 1975, Item 43 above.
61 Leslie , P. H. "On the use of matrices in population mathematics", 1945 *Biometrika* 33: 183-212.
62 Leslie, P. H. "Some further notes on the use of matrices in population mathematics", 1945 *Biometrika* 35:213 – 245.
63 Lotka, A, "Relation Between Birth and Death Rates" in *Science* 1907, 26(253):435-438
64 Schadach, D. "A Classification of Mappings", reprinted in relevant part as Appendix 2 of Ballonoff 1987, Item 44 above,
65 Grimaldi, R. P., *Discrete and Combinatorial Mathematics*, Addison Wesley, Reading Ma., 1989
66 Hildon, P.; Peterson, J.; Stiger, J. On Partitions, Surjections and Stirling Numbers. In *Bulletin of the Belgian Mathematical Society; Belgian Mathematical Society*: Brussels, Belgium; 1994 Volume 1, pp. 713–735.
67 van Bath, B.H. Slicher *The Agrarian History of Western Europe* London: Edward Arnold. 1963
68 Madan, T. N. *Family and Kinship*, Asia Publishing House, Bombay. 1965
69 Madan, T. N. Modern Transformations of Moenkopi Pueblo, University of Illinois Press, Urbana Illinois. 1974
70 Ballonoff, P. "Theory of Lineage Organizations" in *American Anthropologist,* 1983 Vol. 85 no. 1, pp. 79-91]
71 Ballonoff, P. "More on the Mathematics of Rule-Bound Systems", in *Cybernetics and Systems: An International Journa*l, 1995, 26:129 – 132]
72 Ballonoff, P. "Notes Toward a Mathematical Theory of Culture" in *Mathematical Anthropology and Cultural Theory,* Vol 1 No 1, November 2000, http://www.mathematicalanthropology.org
73 Ballonoff, P "Structural statistics: Models relating demography and social structure with applications to Apache and Hopi", 1973, *Social Biology* 10:4 pp. 421-426)
74 Ballonoff, P. "On The Evolution of Self-Awareness" pages 347 – 352 in Trappl (ed.) *Cybernetics and Systems 2000*, Austrian Society for Cybernetic Research, proceedings of the European meetings on Cybernetics and Systems Research 2000
75 Williams, R. J. *Biochemical Individuality* University of Texas Press, Austin, 1956.
76 Zheng, W., Zhao, H., Mancera, E., Steinmetz, L., Snyder, M., "Genetic Analysis of Variation in Transcription Factor Binding in Yeast", *Nature*, published online 17, March 2010,
77 Medawar, P. B. *The Uniqueness of the Individual*, Basic Books, New York, 1958
78 Kasowski, M., Grubert, F., Heffelfinger, C., Hariharan, M., Asabere, A., Waszak, S., Habegger, L., Rozowsky, J., Shi, M., Urban, A., Hong, M., Karczewski, K., Huber, W., Weissman, S., Gerstein, M., Korbel, J., Snyder, M., "Variation in Transcription Factor Binding Among Humans", *Science*, published online 18 March 2010.
79 American Statistical Association pamphlet "Provides Principles to Improve the Conduct and Interpretation of Quantitative Science", 2016, pgs 1 – 3
80 Baily, Ronald, "Most Scientific Finds are Wrong or Useless" *Reason.com*, http://reason.com/archives/2016/08/26/mostscientificresultsarewrongoruse/print

81 Vax, R. M. "Relatives, Molecules, Particles", *Mathematical Anthropology and Cultural Theory*, 2014, Vol. 7 No. 1, pp. 1 – 157.
82 Haldane, J. B. S, A mathematical theory of natural and artificial selection. Part I, *Transactions of Cambridge Philosophical Society* 1924, 23: 19 – 41
83 Lumsden J. L and E. O. Wilson, *Genes, Mind and Culture*, Harvard University Press, Cambridge, 1981.
84 El Guindi, F. and D. Read "Westermarck Hypothesis Reconsidered: A Comment on Kushnick and Fessler", *Current Anthropology*, Vol. 53, No. 1 (February 2012), pp. 134-135, Published by: The University of Chicago Press on behalf of Wenner-Gren Foundation for Anthropological Research http://www.jstor.org/stable/10.1086/663576?origin=JSTOR-pdf
85 Read, D. "Incest Taboos and Kinship: A Biological or Cultural Story?" *Reviews in Anthropology*, 43:150-175, 2014. ISSN: 0093-8157 print/1556-3014 online DOI: 10.1080/00938157.2014.903151
86 Ballonoff, P. "MV-Algebra for Cultural Rules". *Int. J. Theor. Phys*. 2008, 47, 223–235.
87 Ballonoff, P. "Some Properties of Transforms in Culture Theory". *Int. J. Theor. Phys*. 2010, 49, 2998–3004.
88 Ballonoff, P. "Restatement of the Theory of Cultural Rules" *Mathematical Anthropology and Cultural Theory*, 2008, Vol. 2 No. 2, pp. 1 – 23.
89 Dalla Chiara, M., M.R, Guintini and R. Greechie, *Reasoning in Quantum Theory, Sharp and Unsharp Quantum Logics*, Kluwer Academic Publishers, Dordrecht. 2004
90 Isham, C. J. "Quantum Logic and Decohering Histories", *arXiv:quant-ph*/9506028. 1995
91 Isham, C. J. *Lectures on Quantum Theory* 1995, Imperial College Press. 1995
92 Sorkin R.D. "Space time and causal sets" in *Proc. Conf. SILARG VI*, 1990 Cocoyoc, Mexico
93 Sorkin R.D. and D.P. Rideout "A Classical Sequential Growth Dynamics for Causal Sets" in *Phys. Rev. D* 61 024002 (2000) E-print archive gr-ge/9904062.
94 McElreath, R. and R. Boyd *Mathematical Models of Social Evolution*, 2007, University of Chicago Press, Chicago.
95 Gintis, H. *Individuality and Entanglement* 2016, Princeton University Press, Princeton
96 Washburn, S. L. "Human Behavior and the Behaviour of Other Animals", *American Psychologist*. 1978, 33: 405-418.