

**REFLECTIONS ON THE
ALGEBRAIC REPRESENTATION OF KINSHIP STRUCTURE**

J. M. CARGAL
REVIEWS EDITOR, THE UMAP JOURNAL
JMCARGAL@GMAIL.COM

**COPYRIGHT 2017
ALL RIGHTS RESERVED BY AUTHOR**

SUBMITTED: JUNE 6, 2017 ACCEPTED: SEPTEMBER 15, 2017

**MATHEMATICAL ANTHROPOLOGY AND CULTURAL THEORY:
AN INTERNATIONAL JOURNAL
ISSN 1544-5879**

REFLECTIONS ON THE ALGEBRAIC REPRESENTATION OF KINSHIP STRUCTURE

J. M. CARGAL

Abstract: The reflections here are on work done in 1976 and abandoned in 1980. Nonetheless, after forty years the author may have forgotten the ethnography, but he could not help but reflect on the algebraic aspects, despite himself.

History

The algebraic treatment of marriage systems began with André Weil's appendix [1969] to Claude Lévi-Strauss's landmark *The Elementary Structures of Kinship* [1969]. Note that both were initially published in 1949. This appendix was essentially recapitulated in Kemeny, Snell, and Thompson's [KST] text (1966), a text which was important to bringing discrete mathematics into the modern curriculum. I will refer to that work as *KST*. In August of 1976, essentially in one day, I wrote a paper on the same subject—I did not know what I was doing. That paper [Cargal, 1978] was published in 1978 and I will refer to it as *my paper*. My purpose in this note is to reflect further on the mathematical issues of this area. I will concentrate on the marriage structure of the Kariera because their marriage structure is simple but not trivial.

On Group Theory

There are two immediate problems with the work of Weil and KST.

1. They avoid explicit use of group theory language in order to cater to their intended audiences. This was necessary for their purposes but nonetheless diminished precision.
2. Weil and later KST make use of marriage "types". Two people who can marry one another are of the same marriage type. This does not seem to be a useful tool. Given the total symmetry of the marriage structures addressed, one could view all the marriages as being of the same "type". The only differences are in the labels, which usually correspond to the clans. Men of a particular clan can marry women of a particular clan which happens to be different from their own—that is one of the axioms. Harrison C. White, in his book [1963] dispenses with "types" in his axioms. In my paper I restated the axioms from White (which is useful as his book must be very hard to find) and then I gave them again rephrased as I saw fit.

The thing to note in the treatment of marriage systems by Weil and subsequent researchers is that group theory is not merely a tool for studying the structures, but the structures themselves are rendered as groups. Let us look at the Kariera. The structure of the Kariera is given in the Figure 1 below.

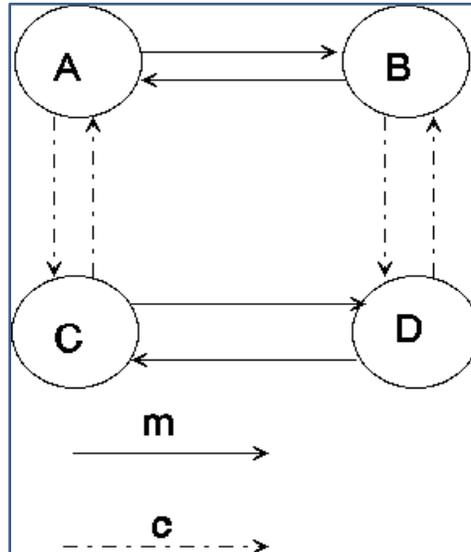


FIGURE 1. THE KARIERA MARRIAGE STRUCTURE

In this graph there are four clans that we label A, B, C, and D. The solid arrows indicate where men get their wives. Following a solid arrow against the indicated direction shows where the women get their husbands. The dotted arrows indicate where the men have their children. Following a dotted arrow against the indicated direction shows the clan of the father. For example, a man of clan D gets his wife from Clan C and has children in clan B. The graph is in fact the Cayley graph of the Klein-four group, aka $Z_2 \times Z_2$.

To get the group structure we need only regard the lines (or arrows) m and c , as group generators. If we denote the identity as e , then the group is completely described by $m^2 = c^2 = e$, and $mc = cm$. So if we want to find the clan of the child of the husband of a woman in D, we compute $Dm^{-1}c = Dmc = Cc = A$ (multiplication is on the right). However, if we choose clan A as our starting point, i.e. as ego, then we can view A as Ae . Then B is Am , C is Ac , and D is $Acm = Amc$. To solve the same problem as above, we replace the D in $Dm^{-1}c$ with Acm giving us $Acmm^{-1}c = Acc = A$. However, if one does the type of calculation a lot, then it becomes natural to leave out the A's as extraneous. The same calculation becomes $cmm^{-1}c = cc = e$ (which is A). In this approach, and this is what I always wound up doing, the elements (in this case clans) become e , m , c , and mc (or cm). It is also easy to go in the opposite direction. In this approach, having identified A with the identity e , then Am , which equals B, functions as the m operator. Like wise C is c and D is $mc = cm$. This becomes quite natural. Using this notation, if we want to know the clan of the father of a wife of a man in the B clan, that is $BBC^{-1} = C$ (yes, this is the correct answer).

Clans

When I wrote my paper I was interested in the ethnology of clans. In this paper I only see them as a partition of a society, and I will come back to this later. In my paper, for an ethnological reason, I decided to look at the subclans by gender. So the A clan would be partitioned into the men of clan A, denoted A1, and the women of clan A denoted A0, and similarly for the other clans.

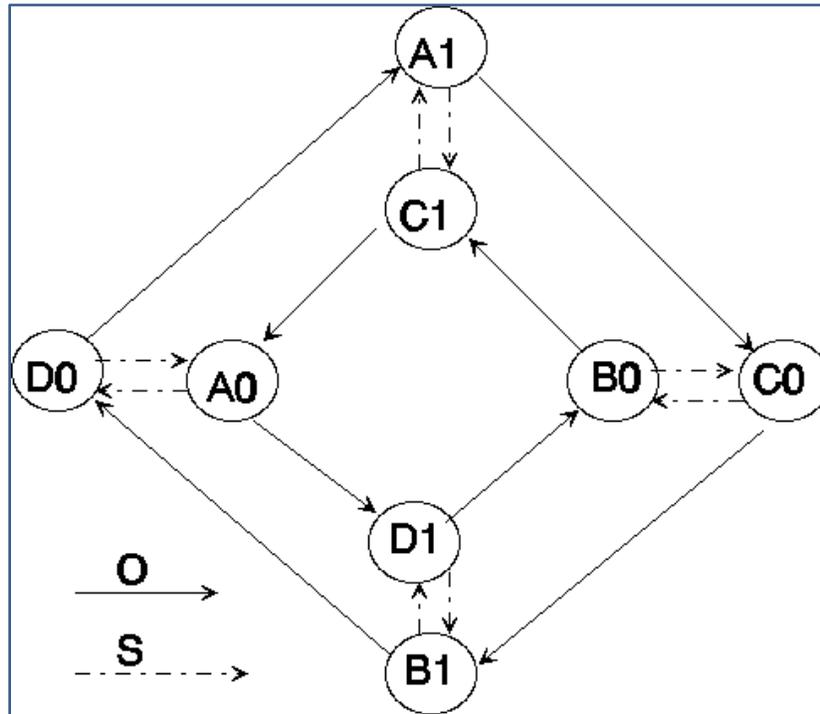


FIGURE 2. THE SUBCLANS OF THE KARIERA

I did the corresponding graph (not shown) using two operators: s for son and d for daughter. It is an interesting graph but it is not the graph of a group. The easy way to see it is not a group is that in the case of a male there are two copies of s going into the node, and the female case has the analogous problem. It was easy though to relabel the edges to get the graph of a group. This graph is shown in Figure 2. Here the O operator refers to the child of the opposite sex and S refers to the child of the same sex. The group is in fact the dihedral group D_4 —the symmetries of a square. It is not commutative. See Figure 2.

If the group in Figure 1 was the homomorphic image of group of subclans then the kernel of the homomorphism would be a normal subgroup consisting of two elements. It is very easy to find and test all five such subgroups and show that there is no such homomorphism. When I wrote my paper I looked at similar structures on all of the marriage systems I could find, and some of them are not mentioned in the paper. Although I did a great many analyses I suspect I could have missed things. More importantly, I did not spend much time on implications of the subclan

structure shown in Figure 2. In any case as I have shown above, the structures themselves correspond to groups. We would like to do more general analyses. For example, we can generalize to groups that act on sets that do not correspond to groups themselves. When a group, G , operates on a non-empty set, S , we have a mapping $G \times S \rightarrow S$. If e is the identity of G and if $a \in G$ and if $x, y \in S$ we can represent the operation as $a \cdot x = y$; another representation is $a(x) = y$. We require that for any $x \in S$ that $e \cdot x = x$, and for any $a, b \in G$, $(ab) \cdot x = a(b \cdot x)$.

Given just this structure, Burnside's counting theorem [Armstrong, 1997] and related theorems apply. Note that the requirements for a group operating on a set can be fulfilled by a semigroup with identity (aka a monoid). How much structure that implies, I do not know. The closest generalization would be maximal closed subsets (or something similar). Quasigroups will give us orbits. In any groupoid operating on elements of a society, we should look for orbits corresponding to certain relationships. Some of these orbits should correspond to clans or identify clan-like structures. My main caveat to that observation is that in identifying clan-like structures, given enough data, statistical techniques may be more effective. For example, high schools are government created entities that partition students into classes which are actually called "classes", e.g. "the senior class" or "the class of 1995". However, high schools spontaneously partition into classes based upon popularity, with the bottom class being the outcasts. Furthermore there are clubs. One person might belong to the French Club and the Chess Club. Another might belong to no clubs. (An interesting question is whether the intersection of the class of most popular students and the Chess Club is null.) To identify spontaneously forming classes one might use statistical techniques and/or various forms of cluster analysis.

The Issue of Associativity

When I did my work in the mid-70s, I was obsessed with the question of tying the group axioms to aspects of the marriage models. I finally realized that the solution was to define a group as a semigroup that is also a quasigroup (i.e. as an associative quasigroup). It is easy to show that this definition is equivalent to the more current definitions of what a group is. (I don't think I realized that this definition corresponded to how Cayley defined groups.) A quasigroup is simply a groupoid, H , such that given any two elements $a, b \in H$, then both equations $a \cdot x = b$ and $x \cdot a = b$ have unique solutions. My thinking was that to generalize the marriage models we could think in terms of quasigroups and jettison associativity. Marriage systems tended to be quasigroups if men (women) got their wives (husbands) from unique clans, and similarly if they sent their children into unique clans.

I probably had things backwards. Associativity is not easy to get rid of. The clan of the son's (daughter's husband) is the same as the clan of the (son's daughter's) husband. It may be useful to get away from the quasigroup's insistence on uniqueness. Consider for example a system with three clans: A, B, and C, such this system men in B get their wives from C and men in C get their wives from B. However, men of A get their wives from A, B, and C. Hence we do not have a quasigroup. A very interesting question is what types of relations or sets of relations imply associativity and what are the implications of associativity?

References

- Armstrong, M. A. *Groups and Symmetry*, Springer, New York, 1988, corrected second printing 1997
- Cargal, J. M. "An Analysis of the Marriage Structure of the Murngin Tribe of Australia." *Behavioral Science*, **23**, May 1978, 157-168.
- Kemeny, J. G., Snell, J. L., and Thompson, G. L. *Introduction to Finite Mathematics*, 2nd ed. Englewood Cliffs, N.J.: Prentice-Hall, 1966.
- Lévi-Strauss, C. *The Elementary Structures of Kinship*. Boston: Beacon Press, 1969.
- Wéil, A. On the Algebraic Study of Certain Marriage Laws (Murngin System). In Lévi-Strauss, C. *The Elementary Structures of Kinship*. Boston: Beacon Press, 1969.
- White, H. C. *An Anatomy of Kinship: Mathematical Models for Structures of Cumulated Roles*. Englewood Cliffs, N.J.: Prentice-Hall, 1963.